Abstract. Let R, S be two rings with unity, M an S-module, and $f: R \rightarrow S$ a ring homomorphism. If the map $M \rightarrow M$, $m \mapsto f(r)m$ is S-linear for any $r \in R$, then M is a representation module of ring R. This condition will be true if $sf(r) - f(r)s \in \mathrm{Ann}(M)$ for all $r \in R$ and $s \in S$. The class of S-modules M, where $sf(r) - f(r)s \in \mathrm{Ann}(M)$ for all $r \in R$ and $s \in S$, forms a category with its morphisms are all module homomorphisms. This class is denoted by $\mathfrak{I}$. The purpose of this paper is to prove that the category $\mathfrak{I}$ is an abelian category which is under sufficient conditions enabling the category $\mathfrak{I}$ has enough injective objects and enough projective objects. First, we prove the category $\mathfrak{I}$ is stable under kernel and image of module homomorphisms, and a finite direct sum of objects of $\mathfrak{I}$ is also the object of $\mathfrak{I}$. By using this two properties, we prove that $\mathfrak{I}$ is the abelian category. Next, we determine the properties of the abelian category $\mathfrak{I}$, such that it has enough injective objects and enough projective objects. We obtain that, if S as R-module is an element of $\mathfrak{I}$, then the category $\mathfrak{I}$ has enough projective objects and enough injective objects.