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Cite as: AIP Conference Proceedings **2330**, 040023 (2021); https://doi.org/10.1063/5.0043223
Published Online: 02 March 2021

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The Ability of Mathematics Education Students to Build Counterexamples in Solving Cyclic Group Problems

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Abstract. This study examines the students' ability to build counterexamples in solving problems related to cyclic groups. This research is an instrumental type case study research involving 109 undergraduate students of Mathematics Education FKIP Lambung Mangkurat University. Students work on one question, which is part of the midterm exam questions in the form of a statement by providing a true or false answer along with the reasons. The results showed that 76 students (69.72%) answered the statement incorrectly and gave a counterexample, 24 students (22.01%) answered that the statement was true and provided proof. The remaining nine students (8.25%) did not provide a true or false answer. The ability to construct a sample can be grouped into four types, namely: (1) no answer; (2) provide correct examples; (3) provide a counterexample but fail to show the statement is false, and (4) provide proof (which is not true) for a false statement.

INTRODUCTION

The importance of proof in mathematics, especially using a counterexample in mathematics has been expressed by many researchers (1,2). Ko & Knuth found that even though students had completed calculus courses, they still experienced difficulties in proving and producing examples (1). Lew & Zazkis examine the mutual influence of proof and use of counterexamples in the evaluation process of the truth of mathematical statements (2). Such evaluation typically involves either the generation of a proof that establishes the claim as true or generation of a counterexample showing the claim to be false. However, these studies had either students produce counterexamples in such a quick manner that little can be inferred about the counterexample generation process (3) or focused on documenting the differences in the number of examples/counterexamples generated rather than the process by which they were generated (4). The importance of building counterexamples in mathematics has not been followed by a teaching and learning process that focuses on honing evidentiary skills and using exemplars.

Meanwhile, Klymchuk states the reasons for challenging students to learn to use examples with the following purposes (1) for deeper conceptual understanding, (2) to reduce or eliminate misconceptions, (3) to advance mathematical thinking, (4) to enhance generic critical thinking skills, (5) to expand the "example space", (6) to make learning more active and creative. Student opinion after learning to use counterexamples was very positive in learning calculus (5). The majority of students said that they found an effective method for using counterexamples which forced them to pay attention to everything in more detail and had to improve their understanding of mathematical concepts (5). Providing students with learning experiences to create will be useful examples for building their "example space".

This problem is also encountered in the Algebra Structure course. Some students have difficulty understanding the course material, cannot identify the subject matter of the course and do not know how to solve some problems in the subject matter of the course. Students are accustomed to answering based on the examples given so that when asked

The 4th International Conference on Mathematics and Science Education (ICoMSE) 2020
AIP Conf. Proc. 2330, 040023-1–040023-6; https://doi.org/10.1063/5.0043223
Published by AIP Publishing. 978-0-7354-4074-6/$30.00

to make their examples, most students have difficulty. One of the reasons is that students have not answered many practice questions given so that they have not mastered many variations of the questions. As a result, when given a question in the form of a statement that has a false value and they have to make a counterexample, for example, most students still experience problems. Many who answered did not match the purpose of the question. As a result, the ability of students to build counterexamples is still lacking.

RESEARCH METHODS

This research is an instrumental type case study research because this research focuses on the ability of students to build counterexamples in solving problems related to cyclic groups (6,7). The subjects of this study were 109 undergraduate students of Mathematics Education FKIP, Lambung Mangkurat University. The subject works on a question which is part of the midterm exam questions form of a statement by giving a true or false answer along with the reasons. If the subject answers correctly, the subject provides proof to prove the truth of the statement. However, if the subject answers the statement incorrectly, then the subject is expected to be able to provide a counterexample. The questions carried out by the subject concerning the cyclic group are as shown in Fig. 1.

Hint: Pay attention to the questions in the form of statements.
If the statement is true, then prove the statement.
If the statement is false, give a counter-examples.

Question:

Each abelian group is cyclic

FIGURE 1. Instruments of Cyclic Group.

RESULTS OF RESEARCH

The results of the study divided into two forms, namely: 1) distribution of student answers, and 2) analysis of student answers in constructing counterexamples in solving questions.

Distribution of Student Answers

Data were collected from proof and examples produced by participants, namely 109 students, with details of 76 students (69.72%) answering the statement incorrectly and giving examples. However, from the examples given, not all of them can show that the statement in the problem is wrong. As many as 24 students (22.01%) answered that the statement was true and provided proof, and the remaining nine students (8.25%) did not give a right or wrong answer or even had no answer. Students' answers are then grouped into four types, namely (1) no answer; (2) provide correct examples; (3) providing a counterexample but failing to show the statement is false, and (4) providing proof (which is not true) for the wrong statement shown in the Table 1.

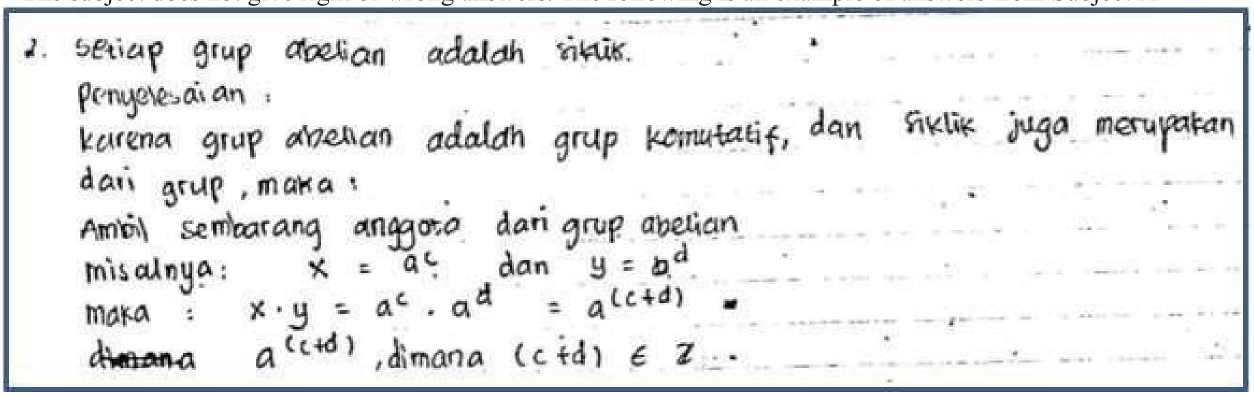
TABLE 1. Four types of student

Answers	Description of Answers
No answers	Not filled, there was the answer but gives no certainty of right or wrong.
Provide correct	The examples given show that the statement in the problem is wrong.
Provide false	The examples provided fail to show that the statement is false.
Providing proof	It was providing false proof to show that the statement is true, Providing proof that is inconsistent with the statements in the questions.

Analysis of Student Answers

This study focuses on analyzing four student answers, namely: (1) not giving correct or wrong answers, (2) correct counterexamples, (3) false examples, and (4) providing (incorrect) proof for a false statement.

(1) The subject does not give right or wrong answers. The following is an example of answers from Subject A

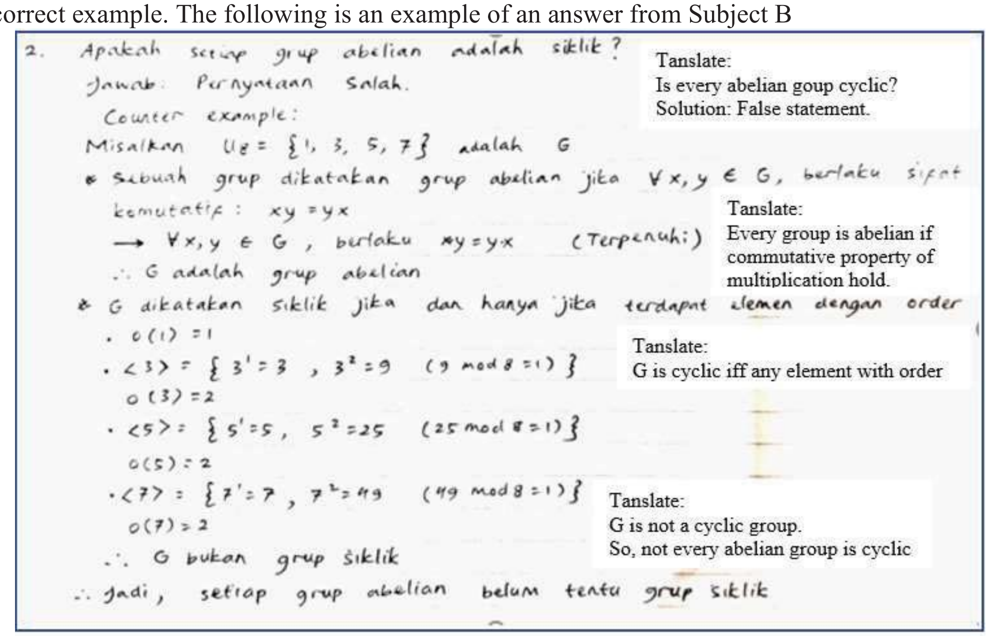

2. Setiap grup abelian adalah siklik.
Penyelesaian :
karena grup abelian adalah grup komutatif, dan siklik juga merupakan dari grup, maka :
Ambil sembarang anggota dari grup abelian
misalnya : $x = a^c$ dan $y = b^d$
maka : $x \cdot y = a^c \cdot a^d = a^{(c+d)}$
~~dimana~~ $a^{(c+d)}$, dimana $(c+d) \in Z$.

Translate: Every abelian group is cyclic.
Solution:
Since abelian group is commutative, and cyclic also a group, therefore:
For any elements of an abelian group.
Let $x = a^c$ and $y = a^d$, so: $x.y = a^c.a^d = a^{(c+d)}$, $(c+d) \in Z$.

FIGURE 2. Answers to subject A

It can be seen in Fig.2 that subject A does not give a true or false answer to the statements in the questions. The subject repeated that what was meant by the abelian group was a commutative group (which was not necessary). Likewise, for cyclic sentences, it is also a group. Furthermore, subject A tries to write down the members of the abelian group that have not been selected or created. As a result, work on subject A was stopped because the workflow from the beginning was unclear.

(2) Give a correct example. The following is an example of an answer from Subject B


2. Apakah setiap grup abelian adalah siklik ?
Jawab : Pernyataan Salah.
Counter example :
Misalkan $U_8 = \{1, 3, 5, 7\}$ adalah G
• Sebuah grup dikatakan grup abelian jika $\forall x, y \in G$, berlaku sifat komutatif : $xy = yx$
→ $\forall x, y \in G$, berlaku $xy = yx$ (Terpenuhi)
∴ G adalah grup abelian
• G dikatakan siklik jika dan hanya jika terdapat elemen dengan order
. $o(1) = 1$
. $\langle 3 \rangle = \{3^1 = 3, 3^2 = 9$ $(9 \bmod 8 = 1)\}$
$o(3) = 2$
. $\langle 5 \rangle = \{5^1 = 5, 5^2 = 25$ $(25 \bmod 8 = 1)\}$
$o(5) = 2$
. $\langle 7 \rangle = \{7^1 = 7, 7^2 = 49$ $(49 \bmod 8 = 1)\}$
$o(7) = 2$
∴ G bukan grup siklik
∴ Jadi, setiap grup abelian belum tentu grup siklik

Tanslate:
Is every abelian goup cyclic?
Solution: False statement.

Tanslate:
Every group is abelian if commutative property of multiplication hold.

Tanslate:
G is cyclic iff any element with order

Tanslate:
G is not a cyclic group.
So, not every abelian group is cyclic

FIGURE 3. Answers to subject B

From the picture above, subject B answers that the statement in the question is false then gives an example of U8 as a group. After determining G, subject B shows that G is an abelian group by showing the commutative properties that apply to G. After that subject B shows that G is a cyclic group if there are elements in G with order 4. Furthermore, subject B looks for the order of each -Each element in G and find that none of the elements in G has order 4. So it can be concluded that G is not a cyclic group. From the examples given, subject B can show that the statement of each abelian group is cyclic is a false statement. Subject B's thought flow is following that the statement can only be false if there is an x that makes P(x) true and Q(x) false. This leads to our next outline for disproof. How to disprove P(x)⇒Q(x). Produce an example of an x that makes P(x) true and Q(x) false.

(3) Giving wrong examples, the following is an example of answers from Subject C

From Fig.4 below, subject C answers that the statement in the question is wrong, then gives an example, namely Group H (Zprima < 9, +). Furthermore, subject C wrote that H was an abelian group and continued to look for the generator of each element in H. Subject C concluded that because no H element made up group H, then H was not a cyclic group. From the answer of subject C, it shows that after finding a set of Z prime which is less than 9, but subject C does not check whether the set with the given operation (addition) is a group. The error has been made at the beginning of the work because H is not a group. However, because subject C thought that H was the group, the subject still proceeded to show that H was not a cyclic group. Of course, it seems that each of the groups constructed by the elements in H is not a group. However, the subject did not realize the error until the end of the work.

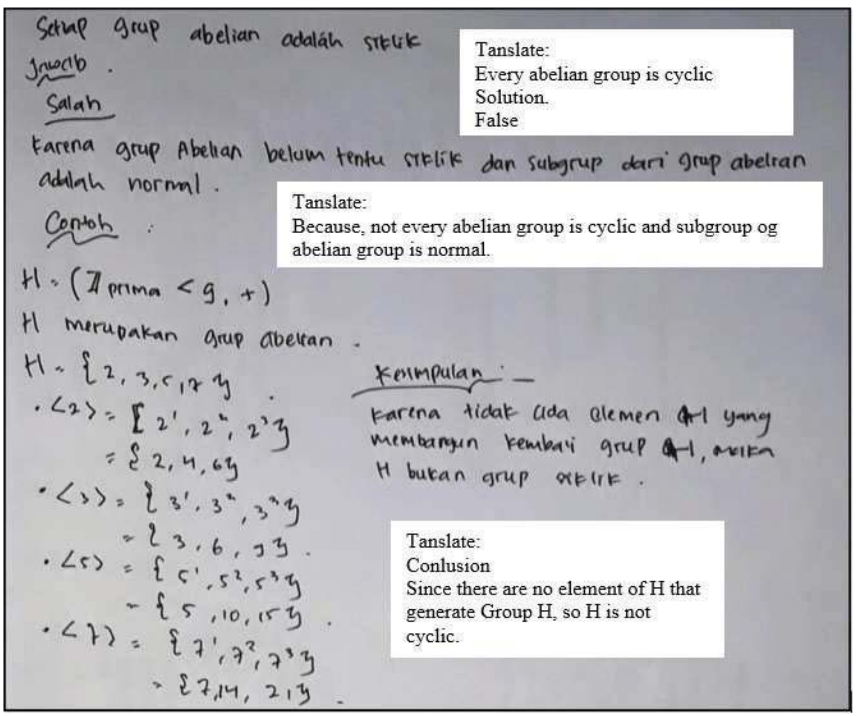


FIGURE 4. Answers to subject C

(4) Giving wrong examples, the following is an example of answers from Subject C

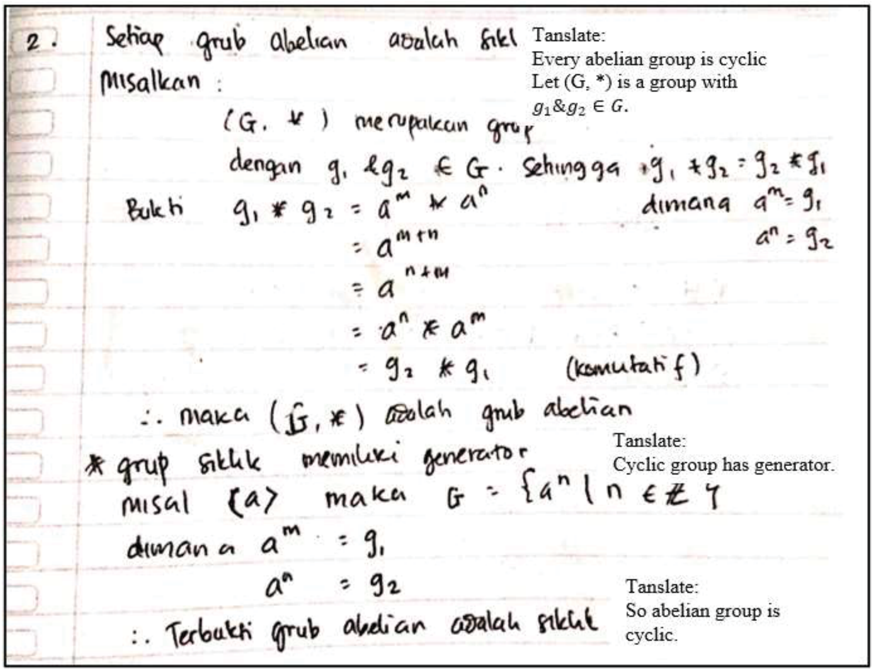


FIGURE 5. Answers to subject D

From the picture above, subject D answers that the statement in the question is true, so it needs to be proven. The subject will show that each abelian group is cyclic. The first step taken is to create a group, namely (G, *). Next step, the subject take the members of G, namely g_1 and g_2 where $am = g_1$ and $an = g_2$. Nevertheless, the subject did not give reasons why taking the form of being as an element of G. Then the subjects D operate$g_1 * g_2$, so we get $g_2 * g_1$ to show that G is an abelian group.

DISCUSSION

The flow of the subject's answer in this paper in building a counterexample is to form an abelian group and show that the group is not cyclic so that it can show the statement in the question is false. However, the results of the study show that not all students can build a counterexample that supports a false statement. The mistakes made by subject C were also made by several other students, that they had not been able to form an abelian group. Subject only considers that the set and the operations defined on the set constitute a group without checking the properties of a group. Even though the subject's line of thinking is correct, it lacks an understanding of the concept of the abelian group.

The condition for a group to be said to be an abelian group is that every two elements of the group are commutative. Like subject C's mistake, several other answers concluded that the examples provided were a noncyclic abelian group. Whereas if further analyzed, the group taken as a counterexample is not abelian because a group is not necessarily an abelian group. After all, not all groups fulfil the commutative properties. Some answers assume that the set taken is an abelian group, but the truth is not proven first. Of course, this line of thinking is not correct because not all sets with certain operations are a group.

Several answers stated that the statement given was true like the answer to subject D so that the subject tried to prove the statement. However, the proof provided is to take any two members of a group and then assume the two members are a commutative generator. In this proof, what is still a question mark is why the two members are assumed to be the generators of the group because not all groups have more than one generator. This error occurs because of the subject's inability to construct a counterexample and a lack of understanding of the concept of groups, abelian groups and cyclic groups.

Lack of understanding of the concepts in the material studied in previous lessons affects the ability to build a counterpart. The production of proof and examples in the results of their research reveals that students have an inadequate understanding of the continuous function to determine the validity of a given statement and produce proof and examples (1). Previous learning experiences also influence the ability of students to build counterexamples; this is in line with the results of research by Lew & Zazkis (2) which states that proving activities that are not explicitly focused on making examples can affect the making of examples, which means that proof efforts can affect the formation student counterexample.

CONCLUSION

The mistake students made in constructing a counterexample was failing to form an abelian group. This mistake is due to a lack of accuracy so as not to re-check the terms of a group. Errors also caused by the lack of understanding of the concept of the group. This problem can be a basis for finding solutions in subsequent lessons by conducting interviews with subjects to dig deeper to find the causative factors. So that in the next lecture, it is hoped that found solutions to overcome student difficulties in building counterexamples.

REFERENCES

1. Ko Y-Y, Knuth E. Undergraduate mathematics majors' writing performance producing proofs and counterexamples about continuous functions. J Math Behav. 2009 Mar;28(1):68–77.
2. Lew K, Zazkis D. Undergraduate mathematics students' at-home exploration of a prove-or-disprove task. J Math Behav. 2019 Jun 1;54:100674.
3. Weber K. How syntactic reasoners can develop understanding, evaluate conjectures, and generate counterexamples in advanced mathematics. J Math Behav. 2009;2–3(28):200–8.
4. Alcock L, Inglis M. Doctoral students' use of examples in evaluating and proving conjectures. Educ Stud Math. 2008 Oct 1;69:111–29.
5. Klymchuk S. Using Counter-Examples in Teaching & Learning of Calculus: Students' Attitudes and Performance. 2012;5(4):31.
6. Fraenkel JR, Wallen NE, Hyun HH. How to design and evaluate research in education. 8th ed. New York: McGraw-Hill Humanities/Social Sciences/Languages; 2012. 642 p.
7. Creswell JW. Educational research: planning, conducting, and evaluating quantitative and qualitative research. 4th ed. Boston: Pearson; 2012. 650 p.