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Critical thinking skill of prospective mathematics teachers in solving the two-dimensional geometry problem

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Critical thinking skill of prospective mathematics teachers in solving the two-dimensional geometry problem

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Abstract. Critical thinking is one of the 21st-century skills that a person needs to have. However, the critical thinking skills of prospective mathematics teachers are relatively low. This study analyzed the critical thinking phase of prospective mathematics teachers in solving a two-dimensional geometry problem, i.e. problem analysis, exploration, drawing conclusions, clarification, and resolution phase. The subjects in this descriptive study were 35 prospective mathematics teachers. In the problem analysis phase, while some prospective mathematics teachers were able to identify what was known or asked in detail, some others were not. In the exploration phase, some prospective mathematics teachers presented ideas in solving problems in one way while others did it in various ways. The subject, who presented the idea of solving the problem in only one way, gave the final conclusion immediately. However, the subject that presented problem-solving in several ways and reviews from various perspectives can provide clarification of the problem and even convey resolution to the problem. This clarification and resolution arose because of cognitive conflicts that occur in the mind of the subject. Cognitive conflict arose because of the contradiction of the solutions to the problem when the subject used problem-solving from a different perspective.

1. Introduction

One of the 21st-century skills that a person needs to be mastered is critical thinking. Thinking critically becomes an aspect that is important in making the national education policy as well as internationally, namely as a means to encourage the citizen to act and play a role in sustainable development [1]. Critical thinking is a mental process that is regulated and plays a role in the decision-making process to solve a problem [2]. Critical thinking including one of the principal that is used to solve various problems in everyday life [3]. In the field of mathematics, critical thinking can enhance creativity by encouraging someone to look for new strategies in solving mathematical problems [4].

Critical thinking skills can not occur randomly or without any effort but structured, deliberate, and trained repeatedly for someone to develop deep thinking [5]. The selection of the right content is appropriate to encourage and develop the skills to think critically [6]. Critical thinking skills that are driven properly can increase mathematical achievement [7]. However, studies that have been conducted show that the critical thinking skills of prospective mathematics teachers are relatively low [8 – 11]. Therefore, there needs to be an in-depth analysis of the critical thinking process of prospective mathematics teachers in order to find solutions to their problems.

This research focuses on two-dimensional geometry. There are several studies that have addressed in solving geometry problems. Solving problems in geometry due to lack of background knowledge and reasoning, misconceptions, and errors in the calculation of basic operations [12]. Meanwhile, problems in solving geometry problems namely (1) only pay attention to the physical appearance of geometric images without paying attention to their geometric properties, (2), the weak combination of properties in geometric images that have been identified with other knowledge needed to solve the problem, (3) generalizing traits that are only in accordance with certain conditions but not in other different conditions [13]. Construction errors in solving two-dimensional geometry problems occur because of illogical construction and construction holes [14]. Illogical constructs occur because prospective mathematics teachers make assumptions that they think are true even though they are substantially incorrect in concept and illogical, while holes of construction occur because of the existence of certain schemes that have not been constructed in the structure of prospective mathematics teacher thinking. However, these studies are limited to uncover the problems associated with solving geometry problems that have been done and have not revealed the role and phases of critical thinking in solving a mathematical problem.

There are five phases of critical thinking skills, which are the trigger event, exploration, drawing conclusions, clarification, and resolution [15]. The trigger event involves the ability to identify the completeness of the premise of a statement and the concept needed to prove the statement. Exploration is the ability to construct meaning and investigate mathematical ideas. Drawing conclusions is the ability to make and decide mathematical ideas inductively or deductively. Clarification is the ability to evaluate and explain, defines the context of mathematical ideas. The resolution, namely the ability to submit / correct mathematical proofs of a statement. Based on the five phases are proposed by Rasiman, researchers reconstructed into a problem analysis, exploration, Draw Conclusions, clarification, and resolution. The purpose of this study is to analyze critical thinking skills of prospective mathematics teachers by examining their expression when confronted with two-dimensional geometry problems in terms of the five phases of critical thinking, which are problem analysis, exploration, drawing conclusions, clarification, and resolution. Descriptions of each phase are presented in Table 1.

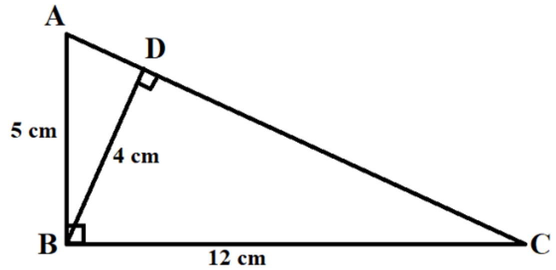
Table 1. Critical Thinking Phase in Solving Geometry Problems

No	Critical Thinking Phase	Description
1	Problem analysis	Identify problems by identifying what is known and asked.
2	Exploration	Solve problems associated what are known from the problem and other initial knowledge that has been owned.
3	Drawing conclusions	Decide on the final solution to the problem given.
4	Clarification	Evaluate the conclusions that have been obtained.
5	Resolution	Filing / fixing problem-solving actions.

2. Method

This research was descriptive qualitative research. The subjects in this study were students of the Mathematics Education Study Program in Faculty of Teacher Training and Education Universitas Lambung Mangkurat at Odd semester 2018/2019. Subjects involved were 35 prospective mathematics teachers. Data collection techniques in this study were carried out by carrying out written tests. The test was done by giving questions in the form of a description of one question about the two-dimensional geometry that students have studied. The test instrument in this study was a modification of As'ari et al. [10] presented in Figure 1.

Look at the figure below.



Determine the area of BCD!

Figure 1. The Test Instrument

Students were instructed to write information that were known and asked about the problem, write down some possible problem solving clearly and in detail, write down conclusions, and write down ideas related to problem-solving that had been done. Data analysis was performed by analyzing the description of student answers based on five phases of critical thinking skills in solving two-dimensional geometry problems namely problem analysis, exploration, drawing conclusions, clarification, and resolution. The analysis was carried out in-depth on the answers of students who were associated with the critical thinking phase. Data analysis techniques in this study included data collection, data condensation, data presentation, and conclusions.

3. Result and Discussion

Results and discussion contain a description of each phase of critical thinking prospective mathematics teacher teachers in solving the two-dimensional geometry problem. The analysis is examined based on five phases of critical thinking skills in solving the two-dimensional geometry problems. The five phases are problem analysis, exploration, drawing conclusions, clarification, and resolution.

3.1. Problem Analyzing Phase in Resolving Two-Dimensional Geometry Problem

Problem analysis is the initial phase carried out by prospective mathematics teachers in solving problems. Problem analysis as a starting point to go to the next stage. Problem analysis as an initial stimulus to critical thinking of subject in addressing a given problem. The subject gives various types of interpretation in analyzing the problem. Subject who are skilled in critical thinking can determine which information are important or not [4].

There are three types of problem analysis presentations given by the subject in solving two-dimensional geometry problems. The first expression is the subject only displays the known lengths. Subject only serving sizes AB, BD, and BC, like that presented in Figure 2. The next presentation that is displayed is the length measurements that are known and the problem in question. This second presentation is complete because it not only conveys the things that are known but also conveys the problem that is asked in the problem. The second type of problem analysis is as in Figure 3.

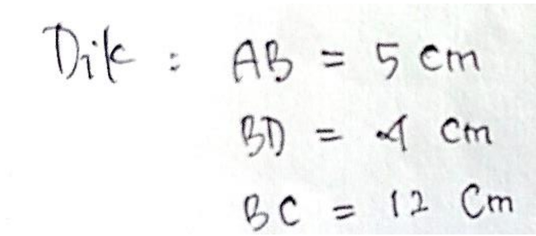


Figure 2. Subject Only Write Known Lengths.

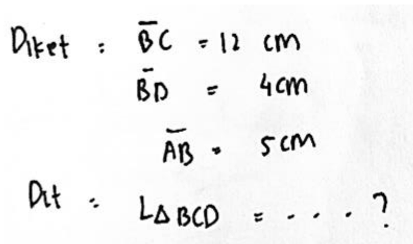


Figure 3. Subject Presenting what is Known and Asked.

The next presentation is the subject conveys the question and the things that are known from the measurements to the meaning of the symbols contained in the two-dimensional geometry, which are the perpendicular symbol. The subject interprets the right-angled symbol by presenting perpendicular lines. The third type of problem analysis is presented in Figure 4.

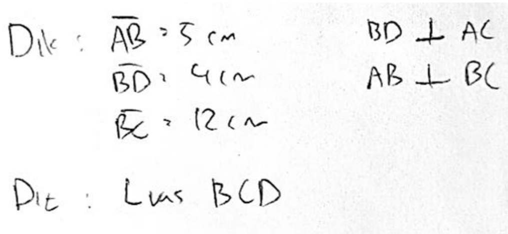


Figure 4. The Known Length, Relationship Between Edges, and The Question

3.2. Exploration Phase in Resolving Two-Dimensional Geometry Problems

Exploration of problem-solving is an advanced phase of analyzing problems. Critical thinking enables subjects to process information in a way that makes sense and prepares them to direct themselves in learning, in case this resolves the problem [4]. At this phase, subject puts their mathematical ideas into is. The ideas are outlined by linking various concepts and procedures that they already have and are associated with information they get from the problem provided. Exploration presented by subject is described in this study.

Three types of problem-solving given by the subject are obtained in this study. The problem solving presented by the subject is presented in Figures 5 and 6.

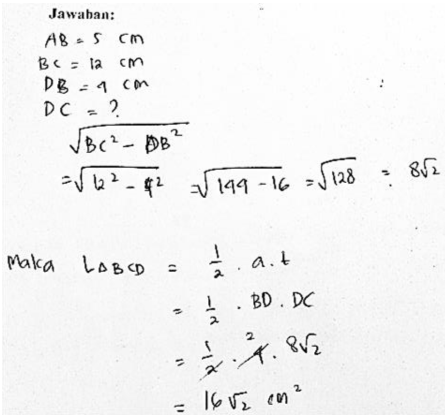


Figure 5. The Solution Presented by Subject.

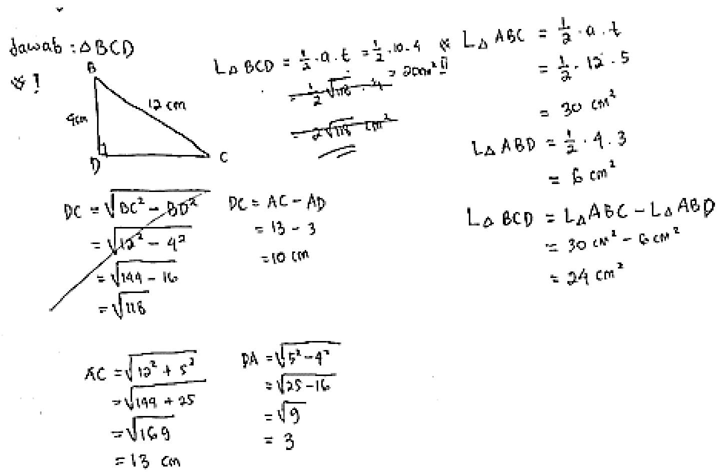


Figure 6. Two Solutions Presented by Subject

Based on Figure 5, subject wrote only one solution to the problem. The subject determines DC through the theorem of Pythagoras and determines the areas were asked. The subject uses information that is already known in the problem as well as the initial knowledge he has without regard to other points of view in solving problems.

In contrast with the answer in Figure 5, subject gives two solutions to the problem presented in Figure 6. The subject has two angles of view different in completing the problem, and both are logical. The first point of view is looking for AC, then DA and DC. After finding DC, the subject determines the area in question. The angle of view of both that the subject determines the area of ABC, ABD, and the last extensive areas BCD by subtracting area ABD of area ABC. The third type of answer done by subjects was writing the three types of solutions, which are presented in Figures 5 and 6. The subject wrote three types of solutions for analyzing the problem from several perspectives.

Of the three types of problem-solving proposed by the subject, there are subjects who only write one type of solution, and there are those who write a combination of more than one type of solution. The difference in the number of resolutions delivered also impacts the subject in providing conclusions, clarifications, and resolutions to the given problem.

3.3. Drawing Conclusion Phase in Resolving Two-Dimensional Geometry Problem

Draw conclusions as the most important part of solving problems. In drawing conclusions, the subject is required to be able to justify the solutions that were done. The conclusions made by subject influence the next phase of action. There are two ways in which subject deliver their conclusions. The conclusion drawn by subjects depends on the number of resolutions made.

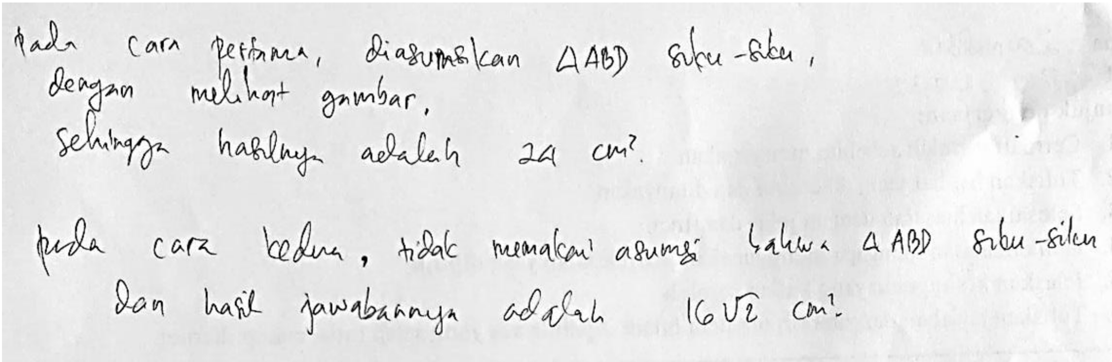


Figure 7. The Conclusions Drawn by Subjects who Resolve Problems in Two Methods of Resolution**.**

Subjects who solve problems in one way directly draw conclusions from the results of their solutions. Figure 7 shows the results of subjects who wrote more than one problem-solving. The subject presents an explanation of the conclusions and resolutions made. Subjects who write more than one conclusion experience cognitive conflict because they find different results when solving problems in different ways. Cognitive conflict triggers subjects to provide clarification.

3.4. Clarification Phase in Solving Two-Dimensional Geometry Problems

Clarification is the ability to provide an explanation of the conclusions drawn. Clarification depends on the conclusions conveyed by subjects. Clarification arises as a result of the conclusions written by subjects. Clarification submitted by subjects was caused by cognitive conflict that occurs after the conclusion. The distribution of subjects' clarification is presented in Figures 8, 9, 10, and 11.

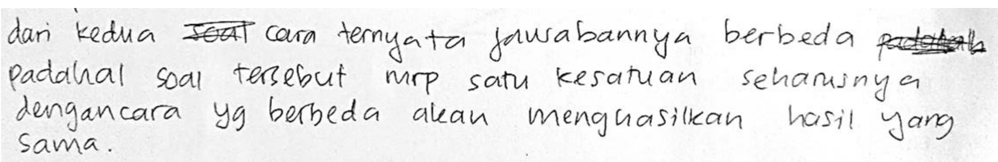


Figure 8. Type 1 Clarification.

Clarification of type 1 is given the students showed their conflict cognitively. Conflict is triggered because the way that has been done to resolve the problem give results that differ. The subject assumes, should give the same results when solving problems from the same figure even if solved in various ways.

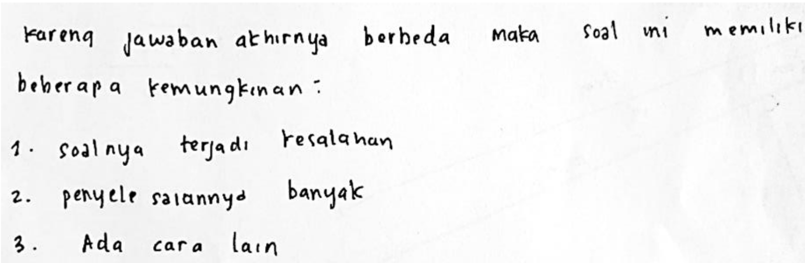


Figure 9. Type 2 Clarification.

Type 2 of clarification shows that the subject justifies the problem given. Justification arises because subjects find different results when solving problems with different methods. The first justification is that the subject considers the given problem wrong. The second justification is that the subject assumes that the given problem has many solutions, even though this assumption is not logical. The third justification is that the subject considers the problem, given the potential to be solved in many ways.

Figure 10. Type 3 Clarification

Type 3 of clarification focuses on the geometric figure of the problem given. The subject clarified that there was an error in placing the right angle (∠BDC). The incorrect placement of right angles results in finding different results with different solutions from different points of view.

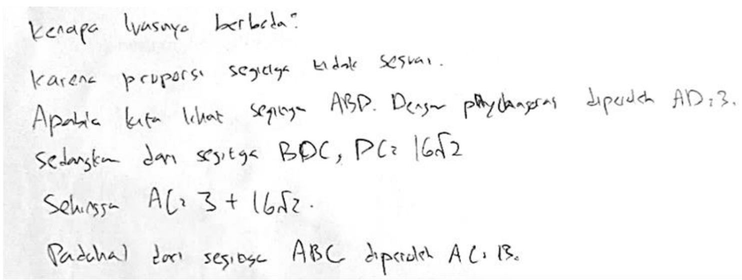


Figure 11. Type 4 Clarification.

Clarification of type 4 is given by the subject is the assumption that the proportion of the size of the triangle is given in the figure is not appropriate. When using the Pythagorean theorem, AD = 3 cm of ΔABD is found and DC = $16\sqrt{2}$ cm of ΔBDC is found so that when added together produces AC = $3 + 16\sqrt{2}$ cm. This contradicts the finding AC = 13 cm of ΔABC using the Pythagorean theorem.

3.5. Resolution Phase in Resolving Two-Dimensional Geometry Problems

Resolution is given as part of further thinking when subjects provide clarification. Resolution is the peak when the subject critically thinks because it proposes or corrects the action that should be taken. In this study, there are two types of forms of resolution. The resolution submitted by the subject is presented in Figures 12 and 13.

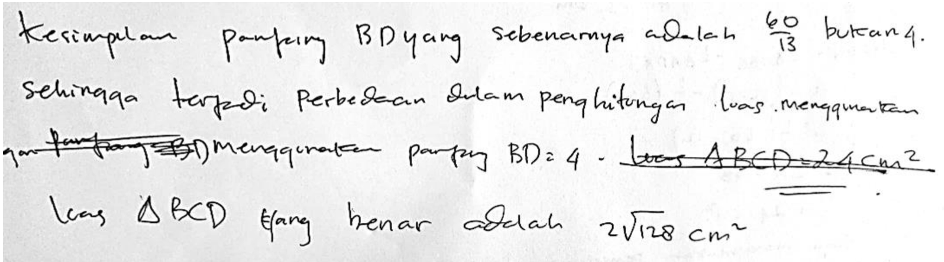


Figure 12. Type 1 Resolution

Type 1 of the resolution presented in Figure 12 shows that the subject gives a correction to the BD. The subject conveys the actual size of BD which is $\frac{60}{13}$ cm, so if given a size of 4 cm, it is not appropriate. The area found by several methods of solution is different because BD = 4 cm is an inappropriate size. The subject gives the correct area calculation result, which is $2\sqrt{128}$ cm^2 with BD = $\frac{60}{3}$ cm.

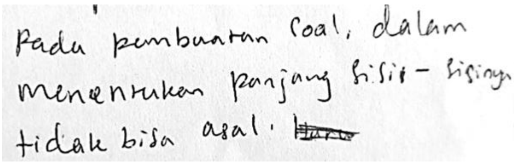


Figure 13. Type 2 Resolution.

Type 2 of the resolution states that a given size cannot be arbitrary. The subject thinks that the given size must match the proportions in the given geometry drawing. The size that is not in proportion will result in the calculation of the requested area will give different results.

3.6. Discussion

There are five phases in which a person can think critically. The five critical thinking phases of a prospective mathematics teacher include problem analysis, exploration, drawing conclusions, clarification, and resolution. The clarification and resolution phase is reached when prospective mathematics teachers experience cognitive conflict in their thought processes. Cognitive conflict occurs because the conclusions obtained are contradicted to the logic of thinking, resulting in clarification and resolution. Clarification and resolution arise when prospective mathematics teacher provides more than one way to solve a problem. By arranging more than one way to solve a problem, prospective mathematics teachers find different results. Clarification and resolution achieved by critical thinkers are in accordance with the statement of Ennis; critical thinking is reflective and reasonable thinking that is focused on making decisions to believe or do something [16]. In addition, clarification and resolution are a form of skepticism. One of the characteristics of a critical thinker is having a skeptic attitude, which is an attitude that can encourage someone to reflect so that it produces the correct conclusions and makes the right decision [17].

As'ari has classified the levels of prospective mathematics teachers' critical thinking skills. These classifications are non-critical thinkers, emergent critical thinkers, developing critical thinkers, and mastering critical thinkers [10]. Noncritical thinkers occur if prospective mathematics teachers do not care about something that should be criticized. Emergent critical thinkers are if prospective mathematics teachers have expressed critical thinking by trying to solve problems. Developing critical thinkers occur if prospective mathematics teachers care about something that needs to be criticized even though the responses given are incomplete or inaccurate. Mastering critical thinkers occur when prospective mathematics teachers always convey critical thinking to produce the best response that is needed or understood. When classified, the phases of critical thinking and classification according to As'ari are presented in Figure 14.

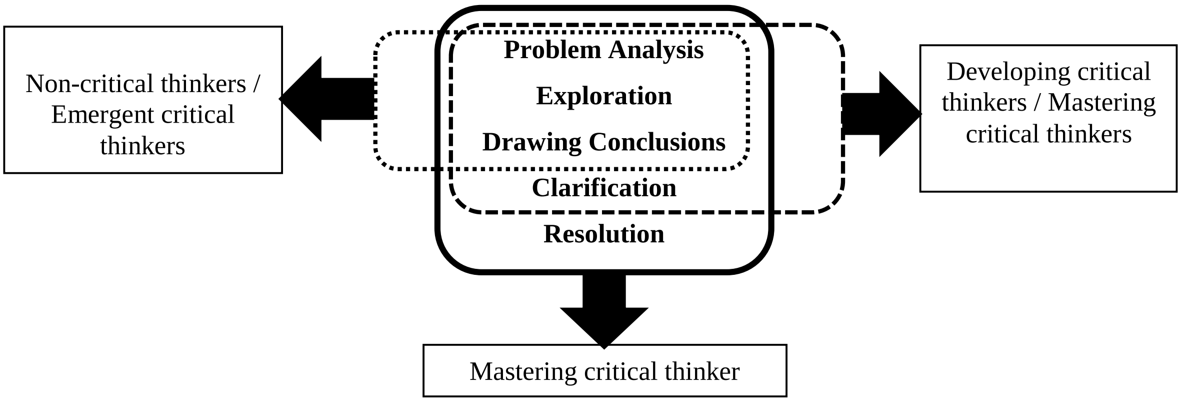


Figure 14. Classification of Critical Thinkers with Critical Thinking Phases.

Based on the phases of critical thinking in Figure 14, when prospective mathematics teachers carry out the phase of problem analysis, exploration to draw conclusions but only have one problem solving, it means that they are still in the category of non-critical thinkers. However, when finished to a conclusion with more than one solution but does not provide clarification, it means prospective mathematics teachers are in the category of emergent critical thinkers. When prospective mathematics teachers reach the clarification phase, a possible category is the developing critical thinker or mastering critical thinker. Developing a critical thinker occurs if the clarification provided is incomplete or inaccurate, but if it is complete and accurate, prospective mathematics teachers fall into the mastering critical thinker category. Prospective mathematics teachers who reach the resolution phase are said to be mastering critical thinkers because, through proper clarification, resolutions will arise. Mastering critical thinkers category for prospective mathematics teachers in accordance with the statement that critical thinking is a mental process that is regulated and plays a role in the decision-making process to solve a problem [2].

4. Conclusion

Prospective mathematics teacher's critical thinking skills in solving problems are involved in five phases, namely problem analysis, exploration, drawing conclusions, clarification, and resolution. In the problem analysis phase, prospective mathematics teachers write in full and also incomplete in identifying what is known or asked in the problem. In the exploration phase, prospective mathematics teachers present the idea of problem-solving in one way or in various ways. Prospective mathematics teachers who convey the idea of problem-solving in one way directly provide final conclusions. However, prospective mathematics teachers who present problem-solving in a number of ways and review from various points of view can provide clarification of problems and even convey resolutions to problems. This clarification and resolution arise because of cognitive conflicts that occur in the prospective mathematics teacher's mind. Cognitive conflict arises because of the contradiction in the solution of the problem when the prospective mathematics teacher uses problem-solving from several perspectives.

Based on the results of the study, there are several suggestions that can be submitted. The researcher provides the suggestions: (1) Prospective mathematics teacher need to be familiarized with the problems that trigger them to think critically, (2) Provision of cognitive conflict can be used as an alternative to practice skills and get used to critical thinking of prospective mathematics teacher, and (3) Prospective mathematics teacher need to be familiarized with open-ended problems in terms of problem solving in order to get accustomed to solving problems from several perspective.

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