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by Sadang Husain

Stability Analysis of a Smoking Behavior Model

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Abstract. This research discussed the mathematical model of smoking behavior. The model will be analogous to an epidemic model which will be divided into several compartments/groups. This research aimed to explain the formation of a mathematical model of smoking behavior, to investigate the equilibrium point, the value of the basic reproduction number, to analyze the stability of the model, then to determine and interpret the numerical solutions using the fourth-order Runge-Kutta method. By the results of this research, a mathematical model of smoking behavior which consists of three compartments, namely the population of non-smokers, smokers and ex-smokers, was obtained. Based on the model formed the smoke-free equilibrium point and the smoker equilibrium point, then the basic reproduction number was also obtained using the *next generation matrix*. Furthermore, the result of the stability analysis of the smoker-free population was asymptotically stable provided that the basic reproduction number is less than one, while the population was asymptotically stable provided that the basic reproduction number is greater than one. The simulation of the model was presented to support the explanation of the stability analysis of the model using the fourth-order Runge-Kutta method based on the parameters that met the requirements of the stability analysis.

1. Introduction

Tobacco is one of the most dangerous killers in the world. In 2015, the Indonesian Health Development Research Agency showed that there were more than 230,000 deaths due to the consumption of tobacco products each year [1]. Globocan stated that of the total cancer deaths in Indonesia, lung cancer was the first cause of death, which was 12.6% [2]. Based on data from the Rumah Sakit Umum Pusat Persahabatan Jakarta, which is the National referral hospital for lung disease, it was found that 87% of lung cancer cases were related to smoking.

Tirtosastro stated that cigarettes contain about 4,800 chemicals with the main components namely tar, nicotine, and carbon monoxide [3]. Soleh in 2017 said that Nicotine can cause pleasant feelings that make smokers addicted to continue smoking. The more a person smokes, the higher content of the tar, nicotine, and carbon monoxide in his body which can cause various kinds of diseases in the body's organs, including several types of cancer-related to the respiratory tract to the lungs, bladder, disorders of pregnancy, and also diseases related to blood vessels such as heart disease and stroke [4].

The mathematical model of smoking behavior was first introduced by Castillo-Garsow et al in 1997 which categorizes the human population into three classes of individuals, namely

non-smokers, smokers, and ex-smokers [5]. Sharomi et al in 2008 have developed a model by dividing two classes of individual ex-smokers [6]. Then in 2019 Hammadi et al have developed a model by dividing the population into three classes of individuals and representing them with a system of nonlinear differential equations [7]. Then in 2020 Simamarta et al reviewed the model that had been developed by Hammadi et al [8]. In this study, the author discusses the formation of a mathematical model of smoking behavior and then proceeds to investigate the equilibrium point, determine stability and perform numerical simulations from the analysis that has been obtained.

2. Method

Metode Penelitian yang digunakan dalam penelitian dilakukan dalam beberapa tahap yaitu melakukan pembentukan model, analisis model dan menentukan solusi model. The first step is to form a model, determine the equilibrium point of the model [9] and the basic reproduction number using the Next Generation Matrix Method ([10], [11]). The second step is to analyze stability through linearization ([11]–[15]). The third step is to determine the model solution using the fourth-order Runge-Kutta [16].

3. Smoking Behavior Model

3.1   *Model formulation*. The following is a compartment diagram of the spread of smoking behavior
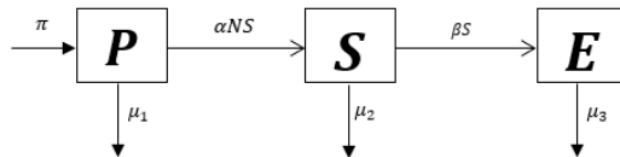

Figure 1. Mathematical Model flow Diagram smoking behavior model.

In this model, the human population divided into four different classes, namely the class of non-smokers, smokers, and ex-smokers. The number of human population at time t is denoted by N(t), while the non-smokers, smokers and ex-smokers are denoted by P(t), S(t) and E(t) respectively. Furthermore, $N(t)$ can be expressed as $N(t) = P(t) + S(t) + E(t)$. The assumptions used in the model are:

1. New individuals due to birth will enter the class of non-smokers, with a constant natural birth rate
2. Non-smokers can become Smokers because of the interaction between Smokers and Non-smokers
3. Individual Smokers are assumed to be able to quit smoking with their own awareness.
4. The death rate of each population class is different. The death rate in the non-smoker population is lower than the death rate for ex-smokers and the death rate in the ex-smoker population is lower than the death rate for smokers.

The mathematical model is given as follows

$$\frac{dP}{dt} = \pi - \alpha PS - \mu_1 P$$
$$\frac{dS}{dt} = \alpha PS - \beta S - \mu_2 S \qquad\qquad (1)$$
$$\frac{dE}{dt} = \beta S - \mu_3 E$$

where

π	:	Rate of population birth
α	:	The rate of transmission of smoking habits due to the interaction of individuals who have never smoked with individual smokers.
β	:	Rate of smokers quitting smoking habits.
μ_1	:	Natural death rate of the non-smoking population.
μ_2	:	The death rate of the smoker population.
μ_3	:	The death rate of the ex-smoker population.

3.2 *Analysis of the Model*. In this model analysis the first step is to determine the equilibrium points. To determine the equilibrium of the system of equations (1) is by creating the system: $\frac{dP}{dt} = 0, \frac{dS}{dt} = 0$ and $\frac{dE}{dt} = 0$

Equation (1) can be written as

$$\pi - \alpha PS - \mu_1 P = 0 \qquad (2)$$
$$\alpha PS - \beta S - \mu_2 S = 0 \qquad (3)$$
$$\beta S - \mu_3 E = 0 \qquad (4)$$

Based on equations (2), (3) and (4), two equilibrium points are obtained, namely

i. The smoker-free equilibrium $T_1 \left(\frac{\pi}{\mu_1}, 0, 0 \right)$

ii. The Smoker-present equilibrium $T_2 \left(\frac{\pi}{\mu_1 R_0}, \left(\frac{\mu_1(R_0-1)}{\alpha} \right), \frac{\beta \mu_1(R_0-1)}{\alpha \mu_3} \right)$

where basic reproduction number $R_0 = \frac{\alpha \pi}{\mu_1(\beta + \mu_2)}$.

In epidemiology, the spread level of an infectious disease is commonly measured by a value called the basic reproduction number (R_0). In this model, if $R_0 < 1$ then the spread of smoking habits gradually decreases and the population will be free from the smoking population and if $R_0 > 1$ then the spread of smoking habits will continue to increase and the smoking population will exist in the population. The basic reproduction number is determined using the next generation matrix method, as follows.

Equation (1) we view into two parts, namely

$$\frac{dS}{dt} = \mathcal{A} - \mathcal{B} \qquad (5)$$

where $\mathcal{A} = [\alpha PS]$, $\mathcal{B} = [\beta S + \mu_2 S]$ and $\begin{pmatrix} \frac{dP}{dt} \\ \frac{dE}{dt} \end{pmatrix} = \begin{pmatrix} \pi - \alpha PS - \mu_1 P \\ \beta S - \mu_3 E \end{pmatrix}$.

Equation (5) is linearized using the Taylor series around the smoker-free equilibrium, we get

$$A = \left(\frac{\partial \mathcal{A}}{\partial S} \right)_{|T_1} = [\alpha P] \qquad (6)$$

and

$$B = \left(\frac{\partial \mathcal{B}}{\partial S} \right)_{|T_2} = [\beta + \mu_2]$$

and

$$B^{-1} = \left[\frac{1}{\beta + \mu_2} \right] \qquad (7)$$

From equations (6) and (7), the next generation matrix is obtained as follows:

$$W = AB^{-1} = \left[\frac{\alpha\pi}{\mu_1(\beta + \mu_2)}\right] \tag{8}$$

The basic reproduction number is the dominant eigenvalue of the W matrix (8), namely

$$R_0 = \frac{\alpha\pi}{\mu_1(\beta + \mu_2)} \tag{9}$$

We refer Theorem 1 to use the basic reproductive number to analyse the stability of the smoker-free equilibrium.

Theorem 1. Consider the system of (1) with the smoker-free equilibrium T_1 and the basic reproduction number R_0 given by (9). If $R_0 < 1$ then T_1 is locally asymptotically stable, but unstable if the $R_0 > 1$.

Proof. Jacobian's matrix on the smoker-free equilibrium $T_1\left(\frac{\pi}{\mu_1}, 0, 0\right)$ can be given by:

$$J(T_1) = \begin{bmatrix} -\mu_1 & \dfrac{-\alpha\pi}{\mu_1} & 0 \\ 0 & \dfrac{\alpha\pi}{\mu_1} - \beta - \mu_2 & 0 \\ 0 & \beta & -\mu_3 \end{bmatrix}$$

$J(T_1)$'s matrix gets a bunch of eigen value, they are $\lambda_1 = -\mu_1$, $\lambda_2 = (R_0 - 1)(\beta + \mu_2)$ and $\lambda_3 = -\mu_3$. All eigenvalues are negative if $R_0 < 1$. Thus, the smoker-free equilibrium is locally asymptotically stable if $R_0 < 1$, but unstable if the $R_0 > 1$.

We refer Theorem 2 to use the basic reproductive number to analyse the stability of the smoker-present equilibrium.

Theorem 2. Consider the system of (1) with the smoker-present equilibrium T_2 and the basic reproduction number R_0 given by (9). If $R_0 > 1$ then T_2 is locally asymptotically stable, but unstable if the $R_0 < 1$.

Proof. Jacobian's matrix on the smoker-present equilibrium $T_2\left(\frac{\pi}{\mu_1 R_0}, \left(\frac{\mu_1(R_0-1)}{\alpha}\right), \frac{\beta\mu_1(R_0-1)}{\alpha\mu_3}\right)$ can be given by:

$$J(T_2) = \begin{bmatrix} -\alpha\left(\dfrac{\mu_1(R_0-1)}{\alpha}\right) - \mu_1 & -\alpha\left(\dfrac{\pi}{\mu_1 R_0}\right) & 0 \\ \alpha\left(\dfrac{\mu_1(R_0-1)}{\alpha}\right) & \alpha\left(\dfrac{\pi}{\mu_1 R_0}\right) - \beta - \mu_2 & 0 \\ 0 & \beta & -\mu_3 \end{bmatrix}$$

$J(T_2)$'s matrix gets a bunch of eigen value, they are $\lambda_1 = -\mu_1$ and other fill up the quadratic polynomial

$$a_0\lambda^2 + a_1\lambda + a_2 = 0$$

where:

$a_0 = 1$

$a_1 = \mu_1 R_0 - \dfrac{\alpha\pi}{\mu_1 R_0} + \beta + \mu_2 = \mu_1 R_0 > 0$

$$a_2 = \mu_1 R_0 (\beta + \mu_2) - \alpha\pi + +\alpha\pi\mu_1(R_0 - 1)$$

$$= \alpha\pi\mu_1(R_0 - 1) + \mu_1(\beta + \mu_2)\left(\left(-\frac{\alpha\pi}{\alpha\mu_1(\beta + \mu_2)} + R_0\right)\right)$$

$$= \alpha\pi\mu_1(R_0 - 1) > 0 \text{ jika } R_0 > 1$$

By using the Routh-Hurwitz theorem [17], all eigenvalues of $J(T_2)$ are negative real parts if $R_0 < 1$. Thus, the smoking-free equilibrium is locally asymptotically stable if $R_0 < 1$, but unstable if the $R_0 > 1$.

4. Numerical Analysis Simulation

In this section, we consider the explicit fourth-order Runge-Kutta (RK4) scheme for solving numerically the non-linear first order ordinary differential equations (ODEs) of our smoking behavior model of Equations (1) with given initial conditions.

4.1 *Numerical Simulation at Smoker-Free Equilibrium.* In this section, we present some numerical simulations to describe solution of the proposed model at Smoker-free equilibrium and Smoker-present equilibrium. Initial conditions of Smoker-free simulation are $P(0) = 100$, $S(0) = 40$, $E(0) = 10$. The parameters on fig.2 are : $\pi = 5, \alpha = 0{,}003, \beta = 0{,}8, \mu_1 = 0{,}05$, $\mu_2 = 0{,}078, \mu_3 = 0{,}06$ and $R_0 = 0{,}0341 < 1$. Furthermore, the numerical solution Equation (1) at the smoke-free equilibrium for 200 iterations is presented in Table 1 below.

Table 1. Numerical Solution at Smoker-Free Equilibrium

Iterasi	t	P	S	E
1	0,5	94,994	29,8418	23,365
20	10	88,091	0,0852	37,591
40	20	92,753	0,0002	20,698
60	30	95,604	0	11,359
80	40	97,334	0	6,234
100	50	98,383	0	3,421
120	60	99,019	0	1,877
140	70	99,405	0	1,030
160	80	99,639	0	0,565
180	90	99,781	0	0,310
200	100	99,867	0	0,170
240	120	99,951	0	0,051
280	140	99,982	0	0,015
320	160	99,993	0	0,005
360	180	99,998	0	0,001
400	200	99,999	0	0,000

The result of numerical analysis of stability of the smoker-free equilibrium which is illustrated in the Figure 2 below:
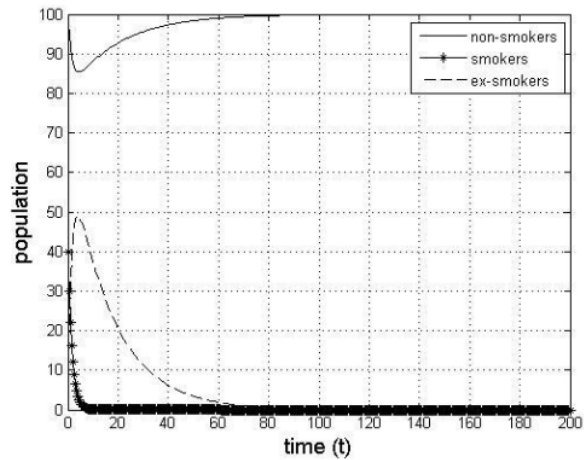
Figure 2. Dynamics of smoking behavior model for smoker-free equilibrium

Based on Table 1 and Figure 2, for an increasing time, the non-smoker population converges towards $99,999 \approx 100$, while the smokers and ex-smokers population towards zero. This indicates that the smoker-free equilibrium $T_1(100; 0; 0)$ is asymptotically stable when $R_0 = 0,0341 < 1$.

4.2 *Numerical Simulation at Smoker-present equilibrium*. In this section, we present some numerical simulations to describe solution of the proposed model at Smoker-free equilibrium and Smoker-present equilibrium. Initial conditions of Smoker-free simulation are $P(0) = 100$, $S(0) = 40$, $E(0) = 10$. The parameters on fig.2 are : $\pi = 10, \alpha = 0,003, \ \beta = 0,8, \mu_1 = 0,05$, $\mu_2 = 0,078, \mu_3 = 0,06$ dan $R_0 = 6,8337 > 1$. Furthermore, the numerical solution Equation (1) at the moker-present equilibrium for 200 iterations is presented in Table 2 below.

Table 2. Numerical Solution at smoker-present equilibrium

Iterasi	t	P	S	E
1	0,5	43,1080	74,0683	33,1351
15	7,5	28,6922	5,5436	120,4623
20	10	34,9481	7,0286	114,8911
40	20	28,7644	9,2466	124,1413
60	30	29,0666	9,8076	126,2218
80	40	29,2801	9,7403	127,6703
100	50	29,2735	9,7204	128,5700
120	60	29,2664	9,7222	129,0552
140	70	29,2664	9,7229	129,3178
160	80	29,2667	9,7229	129,4622
180	90	29,2667	9,7229	129,5416
200	100	29,2667	9,7229	129,5851
220	110	29,2667	9,7229	129,6090
240	120	29,2667	9,7229	129,6221
260	130	29,2667	9,7229	129,6293

280	140	29,2667	9,7229	129,6333
300	150	29,2667	9,7229	129,6354
320	160	29,2667	9,7229	129,6366
340	170	29,2667	9,7229	129,6373
360	180	29,2667	9,7229	129,6376
380	190	29,2667	9,7229	129,6378
400	200	29,2667	9,7229	129,6378

The result of numerical analysis of stability of the smoker-present equilibrium which is illustrated in the figure 3 below:
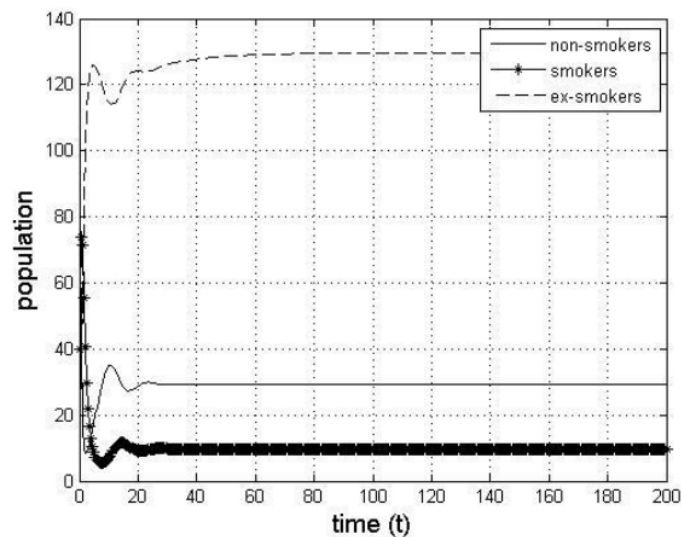
Figure 3. Dynamics of smoking behavior model for smoker-present equilibrium

Based on Table 2 and Figure 3, for an increasing time, the non-smoking population converges towards $29,2667 \approx 29$, the smoking population converges towards $9,7229 \approx 10$ and the ex-smokers population towards $129,6378 \approx 130$. This indicates that the smoker-free equilibrium $T_2(29; 10; 130)$ is asymptotically stable when $R_0 = 6,8337 > 1$.

5. Conclusion
This paper discusses the mathematical model of smoking behavior. The proposed model has two equilibria, namely the smoker-free equilibrium and the smoker-present equilibrium. We find that the properties of the proposed model are completely determined by the basic reproduction number of the model.

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