NIE
NATIONAL INSTITUTE OF EDUCATION
SINGAPORE

Graduate Student Conference

Certificate of Participation

GSC 2017

This is to certify that

Muhammad Ahsar Karim

Participated in the Oral Presentations at the
Graduate Student Conference 2017
held on 28 October 2017 at the
National Institute of Education, Singapore

Ast/P Adrian Kee
Assistant Dean, Higher Degrees by Research
Office of Graduate Studies & Professional Learning
National Institute of Education, Nanyang Technological University

An Institute of NANYANG TECHNOLOGICAL UNIVERSITY

A member of INTERNATIONAL ALLIANCE of LEADING EDUCATION INSTITUTES

UNIVERSITAS LAMBUNG MANGKURAT

M. Ahsar Karim <m_ahsar@unlam.ac.id>

Acceptance of abstract submission for Graduate Student Conference 2017

NIE Graduate Students Conference (GPL) <niegsc@nie.edu.sg> Sat, Sep 23, 2017 at 5:34 PM
To: "m_ahsar@unlam.ac.id" <m_ahsar@unlam.ac.id>

Dear Muhammad Ahsar Karim,

1. We have accepted your abstract submission entitled "**A Comparison of Numerical Performances between Classical and Extended Runge-Kutta Methods for Parameter Estimations of Periodic Type Fuzzy Differential Equations**" for **Oral Presentation** at the Graduate Student Conference 2017.

2. **Here are some Presentation Guidelines for Oral Presentation.**

a) Oral paper presentations will be held in parallel sessions

b) Each parallel session shall consist of a number of paper presentations

c) You will be allocated 15 minutes to present your paper, and 5 minutes for Questions and Answers

d) The usual projection system will be available for your use at the presentation venues

e) The actual presentation timeslot has not been confirmed. We will inform you in due course.

3. **Directions to the Graduate Student Conference 2017 (*for overseas presenters*)**

a) The Graduate Student Conference 2017 will be held in the National Institute of Education (Singapore), an institute in the Nanyang Technological University (Singapore).

b) For the map of Nanyang Technological University, please see http://maps.ntu.edu.sg/maps

c) -For the map of National Institute of Education (Singapore), please see http://www.nie.edu.sg/about-us/visit-us

4. **Should you have any enquiries, please email us at** niegsc@nie.edu.sg

5. **See you at the Graduate Student Conference 2017 on 27-28 October 2017.**

Regards,
Organising Committee
Graduate Student Conference 2017
National Institute of Education, Singapore.

A Comparison of Numerical Performances between Classical and Extended Runge-Kutta Methods for Parameter Estimations of Periodic Type Fuzzy Differential Equations

M. A. Karim[1,2,a)], A. Y. Gunawan[1,b)], M. Apri[1] and K. A. Sidarto[1]

[1]*Department of Mathematics, Institut Teknologi Bandung, Bandung, Indonesia*
[2]*Faculty of Mathematics and Natural Sciences, Universitas Lambung Mangkurat, Banjarbaru, Indonesia*

[a)]Corresponding author: m_ahsar@unlam.ac.id
[b)]aygunawan@math.itb.ac.id

Abstract. In most of real world systems, it may not possible to exactly determine mathematical model structures and parameters of dynamical systems without involving uncertainties in the model possibly either due to limitations of available data, complexity of the networks, and environmental or demographic changes. One of typical behavior that commonly appears in real world systems is a periodic type. In this work, a simple mathematical model describing periodic type, i.e. a harmonic oscillator model, is considered as our subject by assuming that its initial value has uncertainty in terms of fuzzy number. A Fuzzy differential inclusion is chosen as a method to determine solutions. Application of fuzzy arithmetic to the fuzzy model leads us into a set of α-cut deterministic equations. The equations are then solved by two methods: classical and extended Runge-Kutta methods. In contrast to the classical Runge-Kutta method, the extended Runge-Kutta method utilizes new parameters in order to enhance the order of accuracy of the solutions using evaluations of both function and its first derivative. We demonstrate how to estimate parameters of the harmonic oscillator model using both methods to our generated fuzzy simulation data. We finally compare numerical performances for both methods.

NIE Graduate Students Conference 2017 (GSC 2017), 27-28 October 2017

A COMPARISON OF NUMERICAL PERFORMANCES BETWEEN CLASSICAL AND EXTENDED RUNGE-KUTTA METHODS FOR PARAMETER ESTIMATIONS OF PERIODIC TYPE FUZZY DIFFERENTIAL EQUATIONS

Muhammad Ahsar Karim[1,2,*]

Agus Yodi Gunawan[1], Mochamad Apri[1], Kuntjoro Adji Sidarto[1]

[1]*Department of Mathematics, Institut Teknologi Bandung, Bandung - Indonesia*

[2]*Faculty of Mathematics and Natural Sciences, Universitas Lambung Mangkurat, Banjarbaru - Indonesia*

*Corresponding author: *m_ahsar@unlam.ac.id*

Motivations

Modeling in complex systems is often faced with challenges in terms of **measurement uncertainty**
This is possibly either due to ***limitations*** *of* ***available data, environmental*** *or* ***demographic changes***

⇐ Uncertainty Theory

Initial Value Problem with **Periodic Behavior**
(Harmonic Oscillator Model)

⇐
- Probabilistic
- Linguistic

Periodic Type **Fuzzy Differential Equations (FDEs)**
(Fuzzy Initial Value Problem)

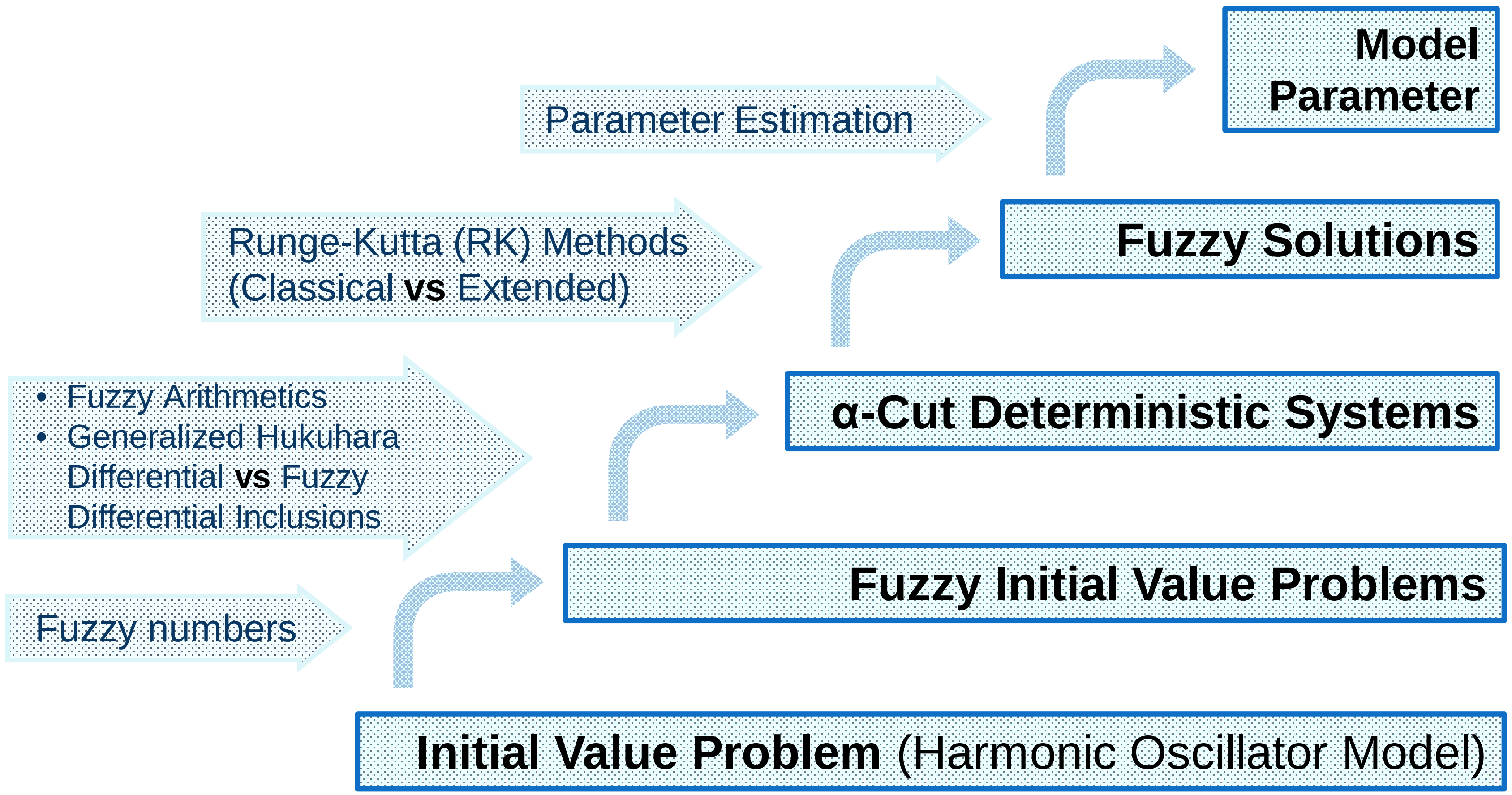
⇐ Fuzzy Theory (Linguistic)

Solving the FDEs

⇐
- Fuzzy Arithmetics
- Fuzzy Differential Types
- Solving Methods

Parameter Estimation

Methodology

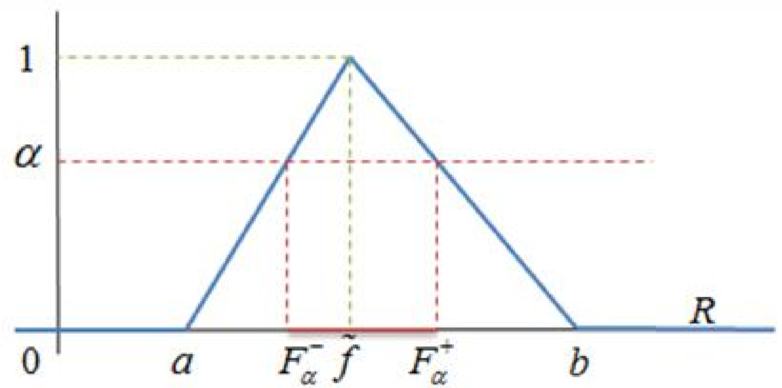


Membership Function

Some of the basic concepts of fuzzy theory, such as fuzzy subset, α-cut, fuzzy number, fuzzy difference, Hukuhara differentiable, and Seikkala differentiable can be found in [6,9,10].

An illustration:
Given a triangular membership function F (one of the forms of fuzzy number) below. The fuzzy number F usually called by "**around** f".

$$trimf_F\left(x,[a,\bar{f},b]\right)=\begin{cases}\dfrac{x-a}{\bar{f}-a} & ;a\le x<\bar{f}\\[2mm] \dfrac{b-x}{b-\bar{f}} & ;\bar{f}\le x<b\\[2mm] 0 & ;x\text{ others}\end{cases}$$



with $a,\bar{f},b\in R$ and $a<\bar{f}<b$.
The α-cut of F shortened by $[F]^\alpha=[F_\alpha^-,F_\alpha^+],\alpha\in[0,1]$.

Arithmetic of fuzzy numbers

Let A and B be fuzzy numbers with α-cuts $[A]^\alpha = [A_\alpha^-, A_\alpha^+]$ and $[B]^\alpha = [B_\alpha^-, B_\alpha^+]$, respectively, and a real number δ.

(a) The sum and the difference of $[A]^\alpha$ and $[B]^\alpha$:
$[A+B]^\alpha = [A]^\alpha + [B]^\alpha = [A_\alpha^- + B_\alpha^-, A_\alpha^+ + B_\alpha^+]$ and
$[A-B]^\alpha = [A]^\alpha - [B]^\alpha = [A_\alpha^- - B_\alpha^+, A_\alpha^+ - B_\alpha^-]$.

(b) The multiplication of $[A]^\alpha$ by δ:

$$[\delta A]^\alpha = \delta[A]^\alpha = \delta[A_\alpha^-, A_\alpha^+] = \begin{cases} [\delta A_\alpha^-, \delta A_\alpha^+]; \delta \ge 0 \\ [\delta A_\alpha^+, \delta A_\alpha^-]; \delta < 0 \end{cases}$$

(c) The multiplications of $[A]^\alpha$ and $[B]^\alpha$:
$[A \cdot B]^\alpha = [A]^\alpha \cdot [B]^\alpha = [\min P, \max P]$;
$P = \{A_\alpha^- B_\alpha^-, A_\alpha^- B_\alpha^+, A_\alpha^+ B_\alpha^-, A_\alpha^+ B_\alpha^+\}$

(d) The division of $[A]^\alpha$ by $[B]^\alpha$, if $0 \notin \operatorname{supp}(B)$:
$[A / B]^\alpha = [A]^\alpha / [B]^\alpha = [A_\alpha^-, A_\alpha^+] \cdot [1 / B_\alpha^+, 1 / B_\alpha^-]$

Fuzzy Differential Equations

- Let R_F be the set of fuzzy numbers, $F, G : (a,b) \to R_F$, $(a,b) \subseteq R$.
  If F and G are **Seikkala differentiable**, then
  $(F+G)' = F' + G'$ and $(kF)' = kF', \forall k \in R$.

- Let $\Im_{\wp}(R)$ be the family of all the fuzzy numbers on R,
  $F : (a,b) \to \Im_{\wp}(R)$ and $F(x) = (f_\alpha^-(x), f_\alpha^+(x))$.
  (a) If F is **Hukuhara Differential** (HD), then $F' = (f_\alpha^-{}', f_\alpha^+{}')$.
  (b) If F is **Generalized HD** (GHD), **then** $F' = (f_\alpha^+{}', f_\alpha^-{}')$.

A HD function is also a Seikkala differentiable.

- A **Fuzzy Differential Inclusion** (FDI)

(i) defined as

$$y'(t) \in F(t, y(t)), y(0) \in \tilde{y}_0$$

(ii) interpreted as

$$y'(t) \in [F(t, y(t))]^\alpha, y(0) \in [\tilde{y}_0]^\alpha; \alpha \in [0,1],$$

where $[F]^\alpha : [0,T] \times R^n \to \Im_{\wp}(R)$ and $[\tilde{y}_0]^\alpha \in \Im_{\wp}(R)$.

If F is continuous and bounded, then all the solutions to FDI are defined and are the α-cuts of the fuzzy solution.

Runge-Kutta (RK) Methods

Let the system of ordinary differential equations:

$$y'(x) = f(x, y(x))$$

Classical RK Method	***Extended RK Method***
The general form of the main function: $$y_{i+1} = y_i + \left(a_1k_1 + a_2k_2 + \ldots + a_nk_n\right)h$$ with the evaluation functions k_i are: $$k_1 = f(x_i, y_i)$$ $$k_2 = f(x_i + p_1h, y_i + q_{11}k_1h)$$ $$k_3 = f(x_i + p_2h, y_i + q_{21}k_1h + q_{22}k_2h)$$ ... $$k_n = f(x_i + p_{n-1}h, y_i + q_{n-1,1}k_1h + q_{n-1,2}k_2h + \ldots + q_{n-1,n-1}k_{n-1}h)$$ where a_i, p_i and $q_{i,i}$ are constants.	The general form of the main function: $$y_{n+1} = y_n + \sum_{i=1}^{m}(hb_ik_{i1} + h^2c_ik_{i2})$$ with the evaluation functions k_{i1} and k_{i2} are: $$k_{i1} = f\left(x_n + \bar{c}_ih, y_n + h\sum_{s=1}^{i-1} a_{is}k_{s1}\right)$$ $$k_{i2} = f'\left(x_n + \bar{c}_ih, y_n + h\sum_{s=1}^{i-1} a_{is}k_{s1}\right)$$ where $b_i, c_i, \bar{c}_i$ and a_{is} are constants. *The f' is approximated by **forward difference** method.*

Comparison of Fourth-order RK Methods

Examples of models with **available analytical solutions**:

Equations	Exact solution
(1). $y' = \frac{1}{80}(20y - y^2); \quad y(0) = 1, x = [0,10]$	$y = \frac{20}{1 + 19\exp(-x/4)}$
(2). $y' = y + x^2 + 1; \quad y(0) = 1, x = [0,10]$	$y = 4e^x - x^2 - 2x - 3$

$$Error_{\max} = \max\{\,|Num_i - Analitic_i|\,\}, i = 1, 2, \ldots, n$$

Eq. 1 (h)	$Error_{\max}$	
	Classical RK4	Extended RK4
0.10	1.64e-08	1.72e-09
0.05	1.03e-09	1.09e-10

Eq. 2 (h)	$Error_{\max}$	
	Classical RK4	Extended RK4
0.10	6.32e-01	9.21e-02
0.05	4.12e-02	5.75e-03

Comparison of Fourth-order RK Methods

Examples of models with periodical type fuzzy differential equations.

Let Van der Pol model in the form of FIVP:

$$\tilde{y}_1' = \tilde{y}_2,$$
$$\tilde{y}_2' = \lambda \tilde{y}_2 (1 - \tilde{y}_1^{\,2}) - \tilde{y}_1,$$
$$\tilde{y}_1(0) = \tilde{y}_{1\_0},\ \tilde{y}_2(0) = \tilde{y}_{2\_0} \in \Im_{\wp}(R), \quad \lambda \in R$$

α-cuts of state variables $[\tilde{y}_1]^{\alpha} = [y_{1\alpha}^-, y_{1\alpha}^+]$ and $[\tilde{y}_2]^{\alpha} = [y_{2\alpha}^-, y_{2\alpha}^+]$,

initial conditions:

$\tilde{y}_{1\_0} = A = \text{trimf}(x,[0,1,2])$ and $\tilde{y}_{2\_0} = B = \text{trimf}(x,[-1,0,1])$,

with α-cuts:

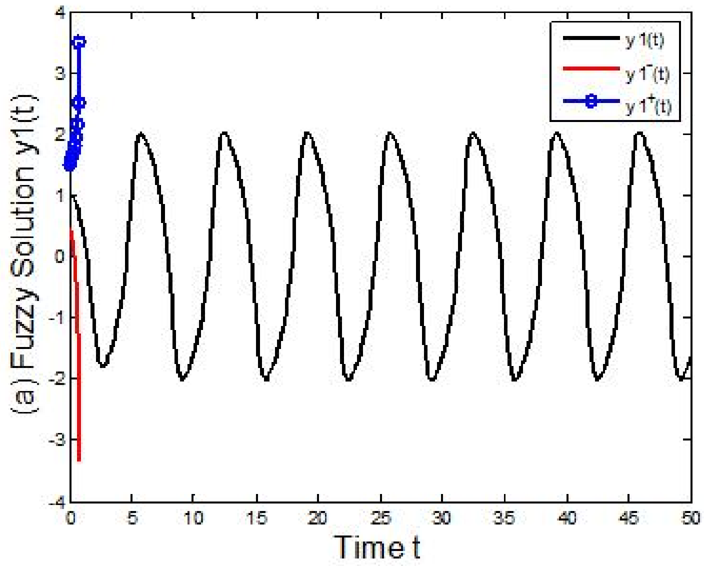
$$[\tilde{y}_{1\_0}]^{\alpha} = [y_{1\_0\alpha}^-, y_{1\_0\alpha}^+] = [0.5, 1.5],$$
$$[\tilde{y}_{2\_0}]^{\alpha} = [y_{2\_0\alpha}^-, y_{2\_0\alpha}^+] = [-0.5, 0.5]$$

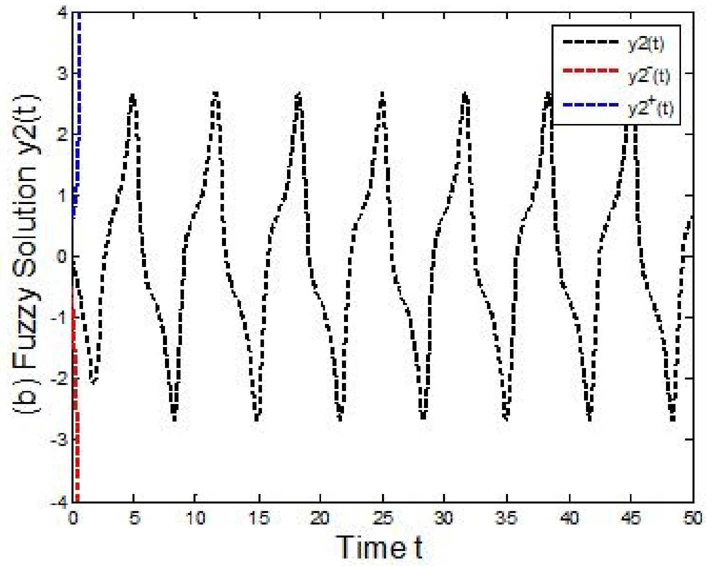
for $\alpha = 0.5$.

Analytical solution is **not available** of the FIVP.

Comparison of Fourth-order RK Methods

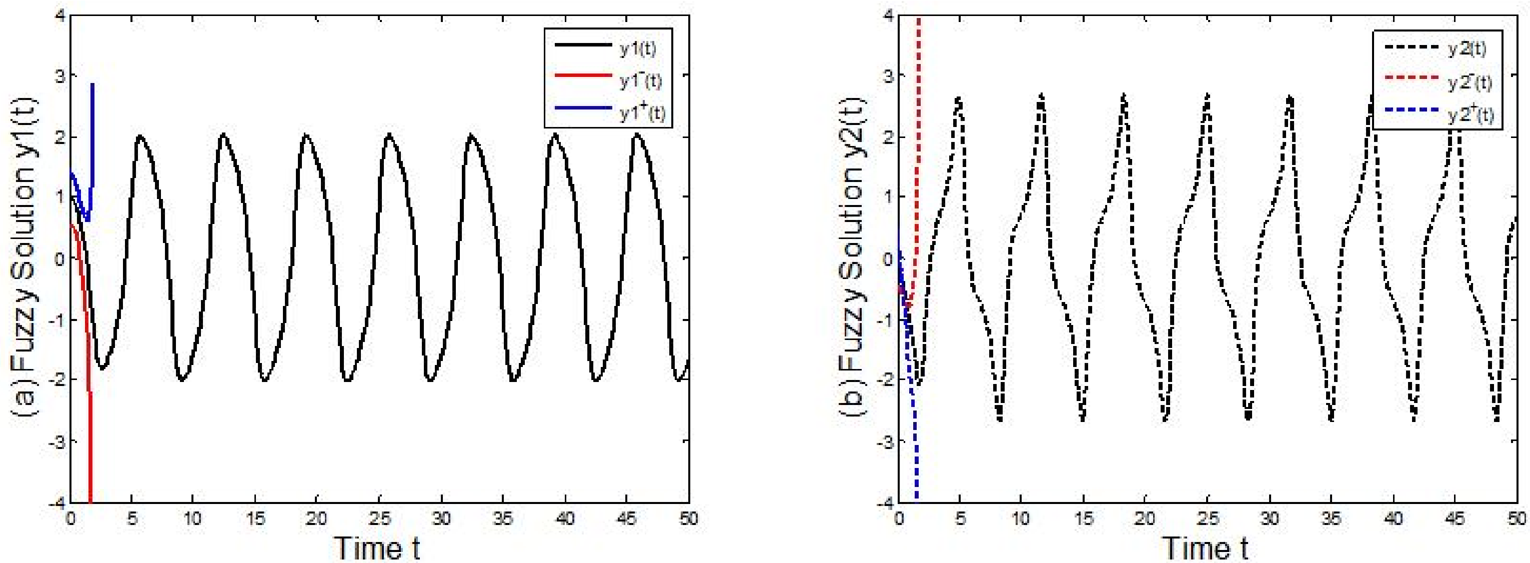
- Numerical solution using HD concept:





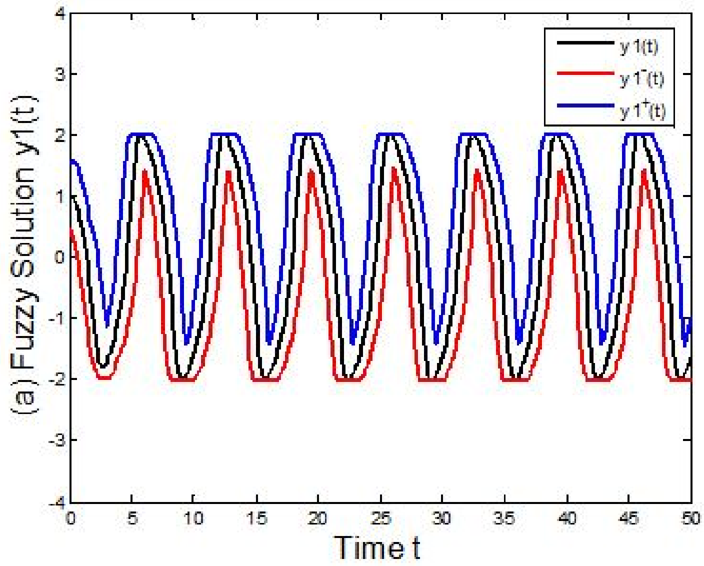
Comparison of Fourth-order RK Methods

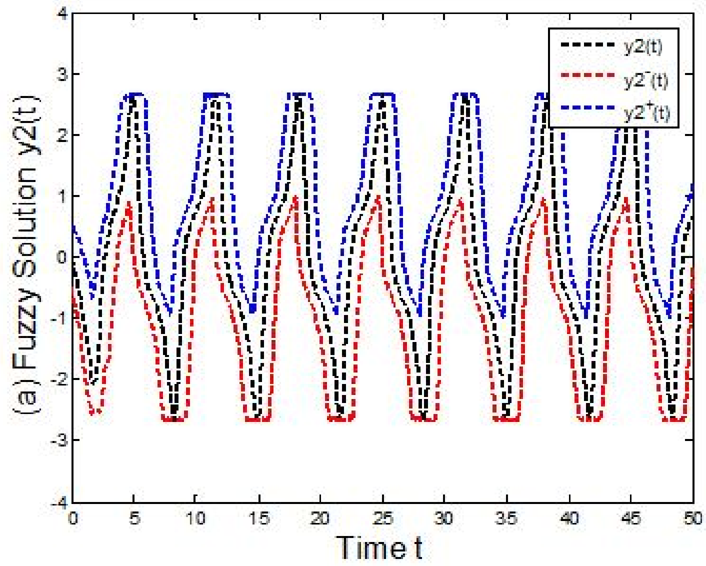
❖ Numerical solution using GHD concept:



Comparison of Fourth-order RK Methods

- Numerical solutions using FDI concept:





Performing comparison of methods, it takes **real data**. But the experiments of this problem are **still going on**.

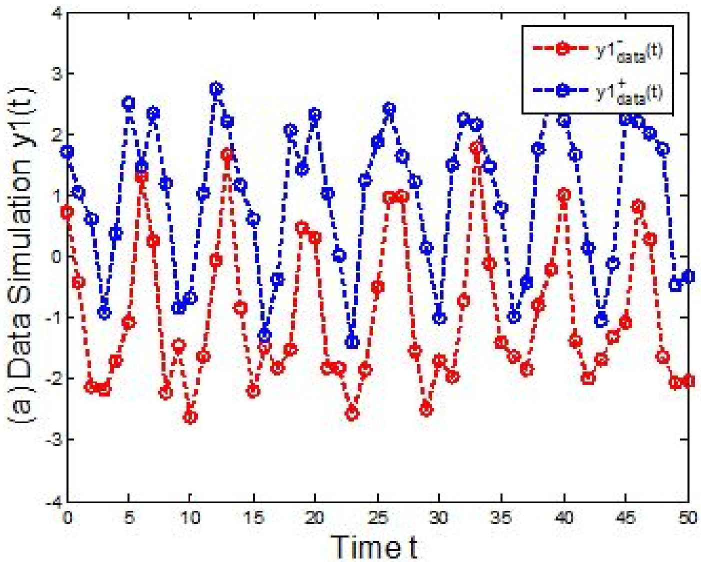
In this presentation we only show the **parameter estimation** process of FIVP Van der Pol model.

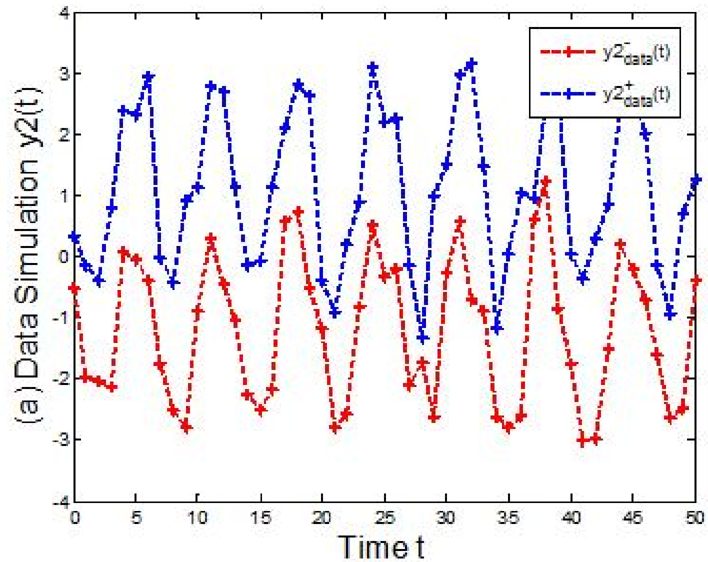
Parameter Estimation

Let the α-cut of **data simulation**:

$$\begin{aligned}[\tilde{y}_{1data}(t)]^{\alpha} &= [y_{1data}{}^{-}_{\alpha}(t), y_{1data}{}^{+}_{\alpha}(t)] \\ [\tilde{y}_{2data}(t)]^{\alpha} &= [y_{2data}{}^{-}_{\alpha}(t), y_{2data}{}^{+}_{\alpha}(t)]\end{aligned},$$

for fixed $\alpha \in [0,1]$ and $t = [0,10]$.





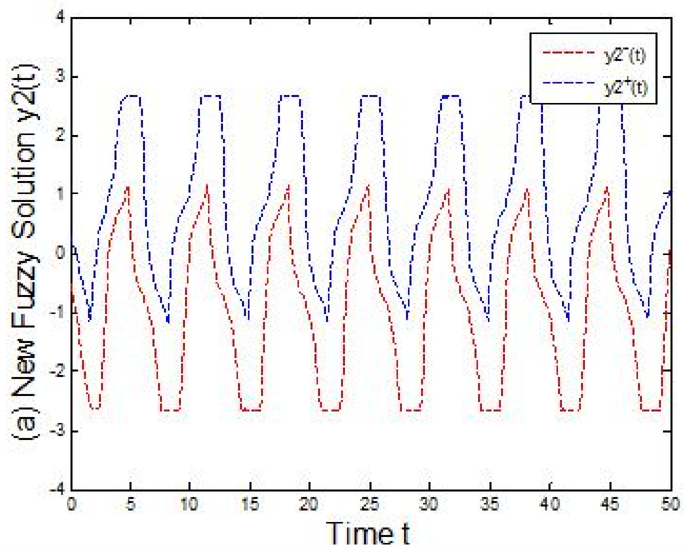
Parameter Estimation

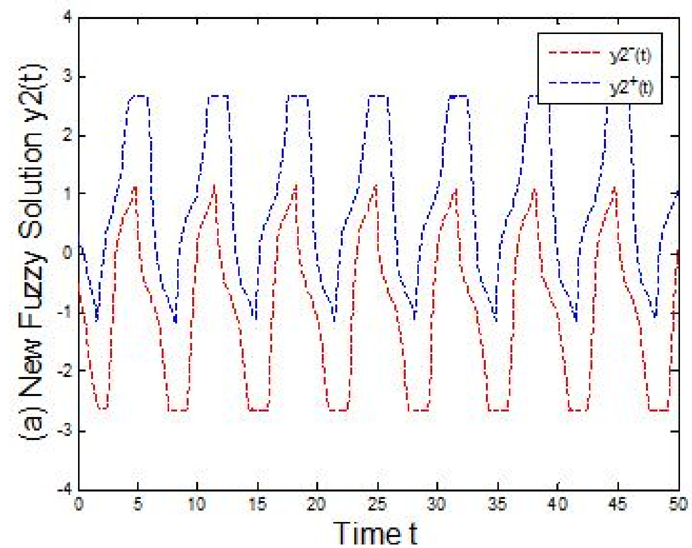
Applying **FDI** concept & **LSQ-Nonlin** method, then optimizing objective function:

$$\min_k \|F(k)\|_2^2 = \min_k \frac{1}{M}\left(\sum_{t=0}^{N-1}\left(y_{1\alpha t}^- - y_{1data\ \alpha\ t}^-\right)^2 + \sum_{t=0}^{N-1}\left(y_{1\alpha t}^+ - y_{1data\ \alpha\ t}^+\right)^2 + \sum_{t=0}^{N-1}\left(y_{2\alpha t}^- - y_{2data\ \alpha\ t}^-\right)^2 + \sum_{t=0}^{N-1}\left(y_{2\alpha t}^+ - y_{2data\ \alpha\ t}^+\right)^2\right)$$

with $M = 4N$ and $k \in [0,2]$.

Producing the parameter k = 0.99999996 & resnorm = 0.22





Conclusions

- Extended Runge Kutta method is significantly better then Classical Runge Kutta method in our some studies. But in the case of periodical type dynamical systems, it is need to studies more intensively.
- In this paper we showed how to choose the appropriate fuzzy method to capture uncertainty for a system having periodical type dynamical system, i.e. Van der Pol model in the form of FIVP. Two method, namely Hd and gHd concepts were not able to capture the oscillations. In contrast, the method using FDI concept was able to capture the oscillations and maintained the uncertainty of the solutions. This led us to apply the FDI concept to estimate the parameters of the FIVP.

Thank You

Muhammad Ahsar Karim
m_ahsar@unlam.ac.id