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at **The 3rd Symposium on BioMathematics (SYMOMATH 2015)**
at Bandung Institut of Technology
on November 4-6, 2015

Chair of SYMOMATH 2015

Symposium on BioMathematics
Symomath 2015
4th - 6th November 2015 / Institut Teknologi Bandung

Dr. Mochamad Apri

Vice-Rector for Research and Innovation
and Partnership ITB

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Decision: Symomath 2015 submission 43

1 message

Symomath 2015 <symomath2015@easychair.org> Thu, Oct 29, 2015 at 5:09 PM
To: Muhammad Ahsar Karim <ahsar.karim@gmail.com>

Dear Muhammad Ahsar Karim, Agus Yodi Gunawan, Kuntjoro Adji Sidarto and Mochamad Apri

Thank you very much for your interest to join Symposium on BioMathematics, 2015, and submitting your abstract entitled: Parameter Estimation in a Fuzzy Dynamical System: A Preliminary Study

It is our pleasure to inform you that your abstract has been accepted for presentation in Symomath 2015.

If you want to publish your paper in the SYMOMATH2015 Proceedings, you are kindly requested to submit the full paper (conforming to the instructions) as soon as possible via symomath2015@gmail.com. Please be reminded that the deadline for the full paper submission is November 1, 2015. The paper will be published in the proceedings by American Institute of Physics (AIP) (indexed in ISI / SCOPUS), please see the author instructions in our website.

The AIP Proceeding paper is now in a single-column format. Thus the double-column format is not longer supported by the publisher. Please notice that the templates that we put in our website until a few days ago were still the old version. But now we have updated the templates with the latest version. You can find them at:
- http://www.math.itb.ac.id/~symomath/contents/aip_cp_2015.zip
- http://www.math.itb.ac.id/~symomath/contents/8x11WordTemplates.zip

Please also fill the attached registration form (if you didn't do it), which can also be downloaded from the conference website and send it via email to:
symomath2015@gmail.com

Details of the conference can be found at http://www.math.itb.ac.id/~symomath/

Thank you very much for your interest in the the 3rd International Symposium on BioMathematics (SYMOMATH 2015) and we are looking forward to seeing you at the Symposium on November 4-6, 2015.

Yours Sincerely
Dr. Mochamad Apri

Conference Chair SYMOMATH 2015

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Parameter Estimation in a Fuzzy Dynamical System: A Preliminary Study

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Abstract

Measurement from experiments often consists of uncertainty. This is possibly due to the limitations of available data taken from experiment. For example, in a biochemical system a number of experiments must be carried out to obtain data accurately. However, in reality, such efforts are limited by, for example, availability of technology, time, and cost. When we model this phenomenon, this uncertainty may lead to unreliable estimation of model parameters. To accommodate the uncertainty, each dependent variable will be assumed to have uncertainty in the terms of fuzzy variables. Application of fuzzy arithmetic to the model leads to *a-cut* deterministic models with extra numbers of equations. We then solve the deterministic equations and estimate the parameters of the system. As an illustration, we apply our method to a simple population growth model.

Keywords: *fuzzy arithmetic, fuzzy variables, a-cut deterministic models*

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PARAMETER ESTIMATION IN A FUZZY DYNAMICAL SYSTEM: A PRELIMINARY STUDY

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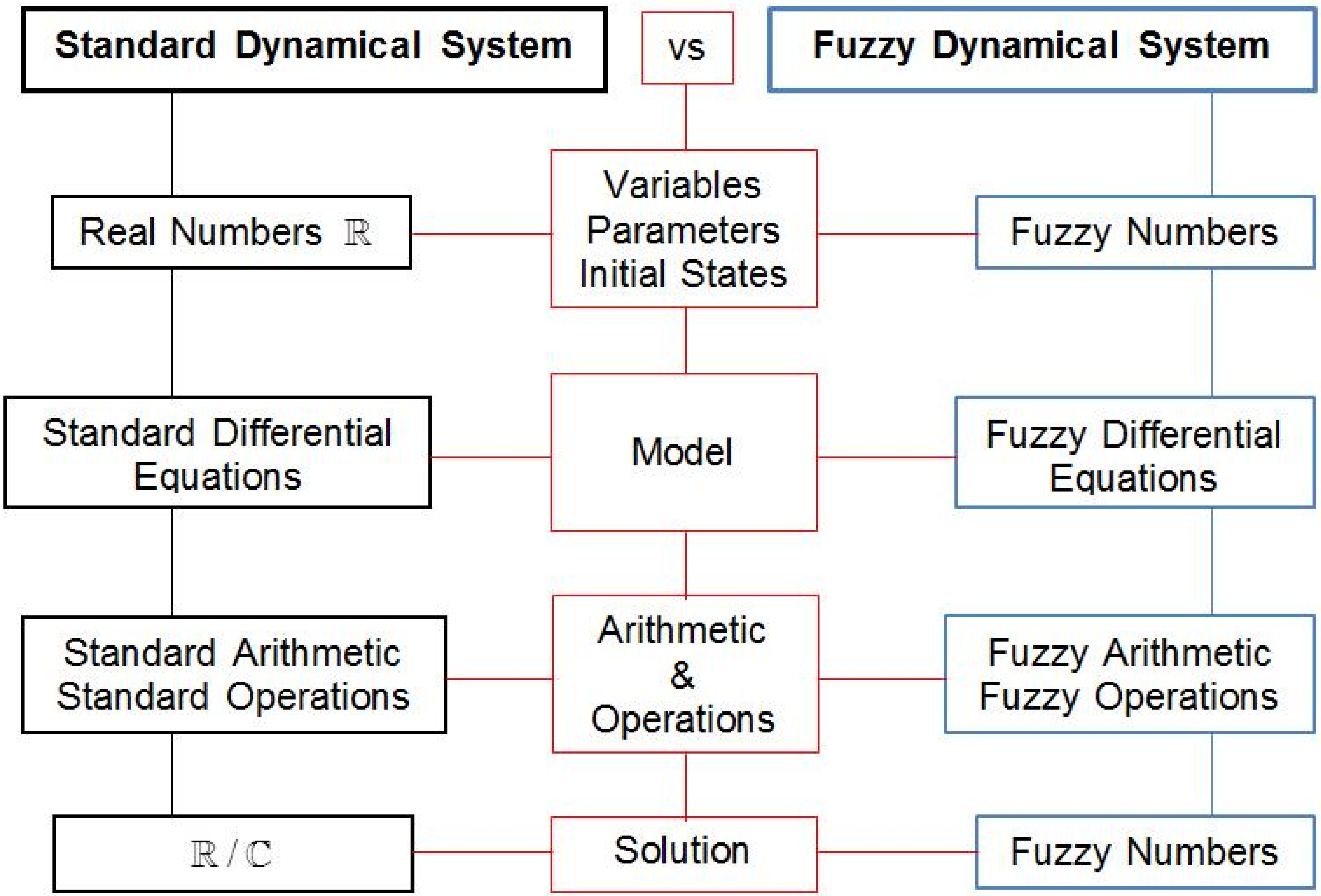
Motivations

- Measurement from experiments often consists of **uncertainty** due to the limitations of available data taken from experiment, which are usually caused by availability of technology, time, and cost.

- To accommodate the uncertainty, we introduce **Fuzzy Concept** in the our variables.

Comparison of Systems



Examples

Classic Dynamical System

- **The *SI* Model**

  The population is homogeneous. That is, each infected individual transmits the disease with the same chance, usually given by the real number β.

- **Predator–Prey**

  The population of predators is assume by homogeneous.

- **Population Growth Model**

$$\dot{Y}(t) = \lambda Y(t)$$
$$Y(0) = Y_0$$

Fuzzy Dynamical System

- **Fuzzy *SI* Model**

  The population is heterogeneous. This is probably caused by the parasite load of infected individuals. So that, β is a fuzzy number.

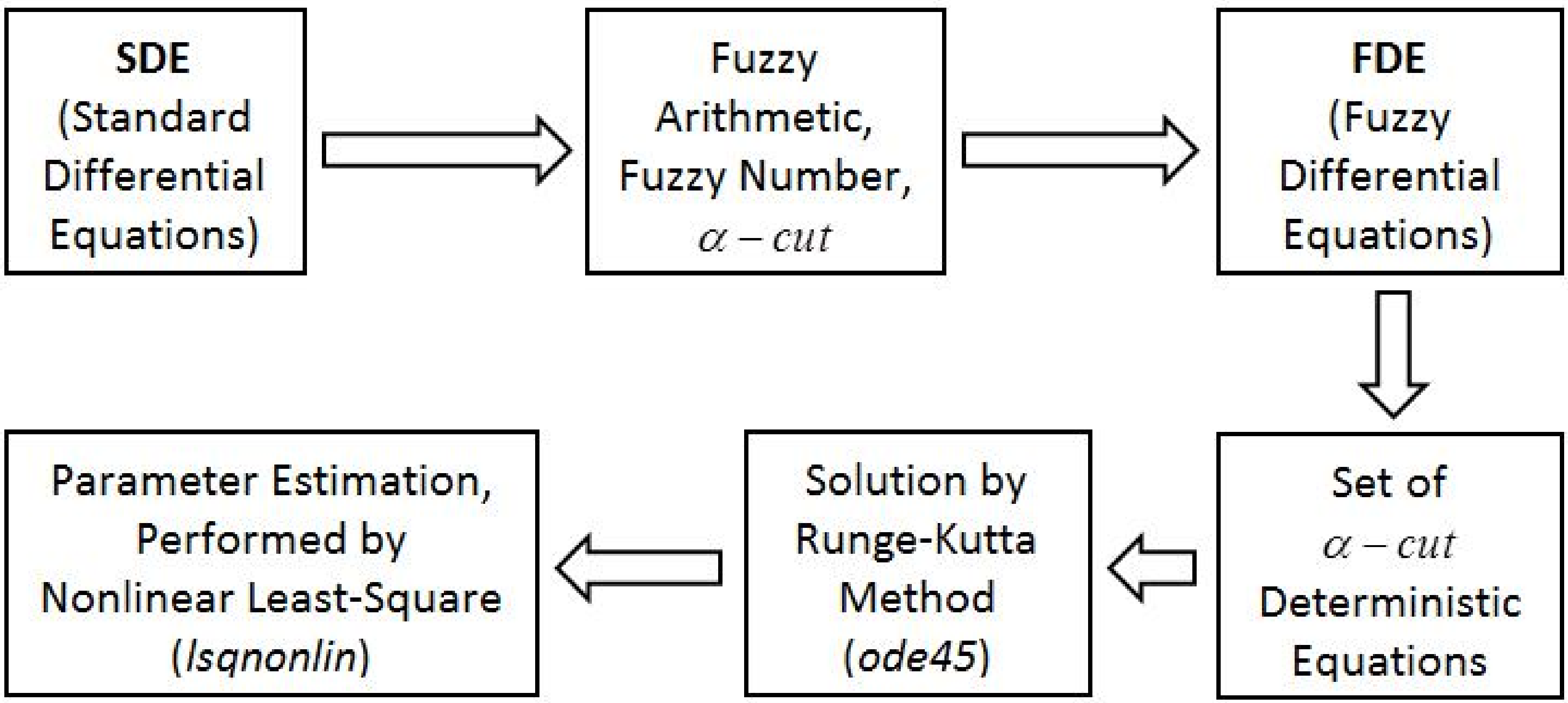
- **Fuzzy Predator–Prey**

  A predator with a certain predatory level can possibly become a prey, depending on environmental circumstances.

- **Fuzzy Population Growth Model**

$$\dot{\tilde{Y}}(t) = \lambda \tilde{Y}(t)$$
$$\tilde{Y}(0) = \tilde{Y}_0$$

Solution Methodology



Basic Concepts of Fuzzy Theory

Definition 1. *Let X be a classical non-empty set and F be a fuzzy subset of X. The $\alpha-cut$ of F, denoted by $[F]^\alpha$ is the set of all elements that belong to a fuzzy set F with at least α degree, that is,* $[F]^\alpha = \{x \in U : F(x) \geq \alpha\}$, $\alpha \in [0,1]$.

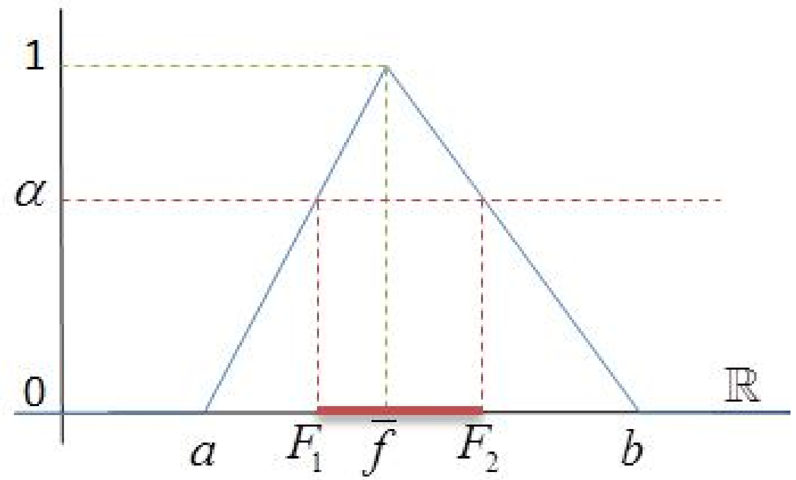
Definition 2. *Let $\mathbb{R}$ be a real number set and F be a fuzzy subset of $\mathbb{R}$. The fuzzy subset F is called by* ***fuzzy number*** *when:*

- *F is normal, that is,* $\exists x \in \mathbb{R} \ni F(x) = 1$.
- $[F]^\alpha, \forall \alpha \in [0,1]$ *are closed intervals of* $\mathbb{R}$.
- *The support of F, that is* $Supp(F) = \{x \in \mathbb{R} : F(x) > 0\}$ *are bounded.*

The collection of all fuzzy numbers F of $\mathbb{R}$ denoted by $\mathbb{R}_F$

Example 1. ***Triangular Membership Function*** is one of the examples of fuzzy number in $\mathbb{R}_F$, which is defined by:

$$trimf_F\left(x,[a,\overline{f},b]\right)=\begin{cases}\dfrac{x-a}{\overline{f}-a} & ;a\le x<\overline{f}\\ \dfrac{b-x}{b-\overline{f}} & ;\overline{f}\le x<b;\\ 0 & ;\ x\ others\end{cases}$$



with $a,\overline{f},b,x\in\mathbb{R}$ and $a<\overline{f}<b$. This fuzzy number is usually called by "around f".

The $\alpha-cut$ of the fuzzy numbers F shortened by $[F]^{\alpha}=[F_1,F_2]$.

Properties of *Alpha-Cut*

Definition 3. *Let A and B be fuzzy numbers with $\alpha - cut$ respectively by $[A]^\alpha = [A_1^\alpha, A_2^\alpha]$ and $[B]^\alpha = [B_1^\alpha, B_2^\alpha]$; and δ a real number.*

- The sum and the difference by A and B:

$$[A+B]^\alpha = [A]^\alpha + [B]^\alpha = [A_1^\alpha + B_1^\alpha, A_2^\alpha + B_2^\alpha]$$

$$[A-B]^\alpha = [A]^\alpha - [B]^\alpha = [A_1^\alpha - B_1^\alpha, A_2^\alpha - B_2^\alpha]$$

- The multiplication of the fuzzy number A by a real number δ:

$$[\delta A]^\alpha = \delta[A]^\alpha = [A_1^\alpha, A_2^\alpha] = \begin{cases} [\delta A_1^\alpha, \delta A_2^\alpha] & ; \delta \geq 0 \\ [\delta A_2^\alpha, \delta A_1^\alpha] & ; \delta < 0 \end{cases}$$

- The multiplication of fuzzy number A by fuzzy number B:

$$[A \cdot B]^\alpha = [A]^\alpha \cdot [B]^\alpha = [\min P, \max P] : P = \{A_1^\alpha B_1^\alpha, A_1^\alpha B_2^\alpha, A_2^\alpha B_1^\alpha, A_2^\alpha B_2^\alpha\}$$

- The division of fuzzy number A by fuzzy number B, If $0 \notin Supp(B)$:

$$[A/B]^\alpha = [A]^\alpha / [B]^\alpha = [A_1^\alpha, A_2^\alpha][1/B_2^\alpha, 1/B_1^\alpha]$$

Fuzzy Differential

Definition 4. *Let* $F:(a,b)\to\mathbb{R}_F;(a,b)\subseteq\mathbb{R}$ *be a fuzzy function. Then* $[F'(x)]^\alpha=\left[(F'(x))_{\inf}^\alpha,(F'(x))_{\sup}^\alpha\right],\forall\alpha\in[0,1],F'(x)\in\mathbb{R}_F$ *is called by* ***Seikkala Derivative*** *of* F *.*

The fuzzy function F in Definition 4 is called by ***Seikkala Differentiable***.

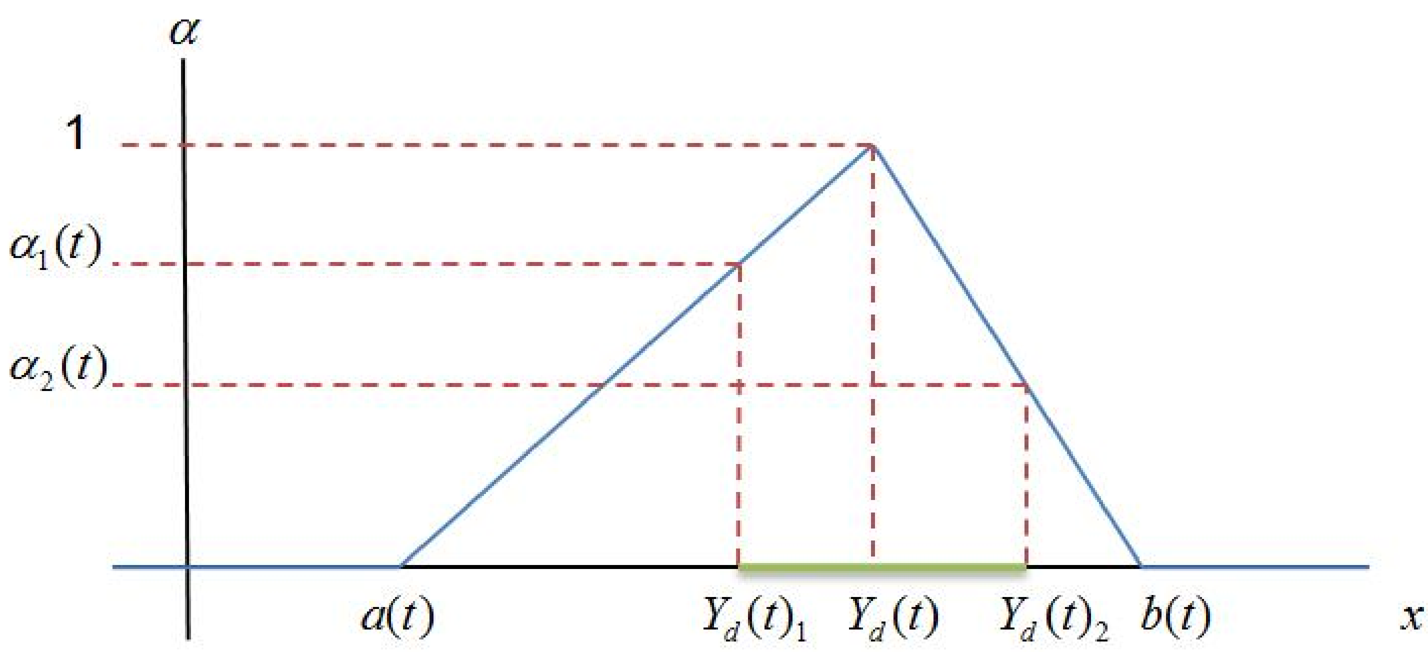
Theorem 1. *Let* $F,G:(a,b)\to\mathbb{R}_F;(a,b)\subseteq\mathbb{R}$ *be a fuzzy functions. If* F *and* G *respectively are Seikkala differentiable, then* $(F+G)'=F'+G'$ and $(kF)'=kF'$, $\forall k\in\mathbb{R}$

Data with contain Uncertainty

No	t	$a(t)$	Sig_1(t)	$Yd(t)$	Sig_2(t)	$b(t)$
1	0	-22.8666	30.9639	78.9006	131.0585	177.7186
2	0.5000	-7.1601	46.2481	96.7233	141.7026	191.8162
3	1.0000	20.6094	68.6863	116.7831	171.5774	223.5580
4	1.5000	13.7802	64.4540	116.1667	169.3465	224.6342
5	2.0000	24.1526	77.7936	129.7609	180.6378	234.8227
6	2.5000	27.2550	80.2698	130.0261	176.7978	227.4361
7	3.0000	39.4503	91.8366	144.1766	200.4010	254.9067
8	3.5000	27.4061	77.8526	123.6375	168.8804	214.1893
9	4.0000	55.3697	102.7149	152.4359	203.5609	254.1903
10	4.5000	33.7547	80.0567	132.2626	190.9452	247.5333
11	5.0000	80.9741	128.2937	177.7863	226.0929	275.1936
12	5.5000	68.2465	117.7964	167.9423	211.9465	256.9079
13	6.0000	85.9459	139.4443	194.1527	259.0665	322.6224
14	6.5000	85.5241	132.1746	181.2587	229.3539	277.8325
15	7.0000	100.2094	156.2633	207.9933	257.1269	314.8294
16	7.5000	122.1799	175.3859	234.0147	277.1774	320.3205
17	8.0000	70.2141	123.5558	178.4951	229.3591	282.4381
18	8.5000	97.5930	142.5861	190.2744	244.3813	297.0071
19	9.0000	109.7736	154.6846	200.8916	251.7183	300.9603
20	9.5000	156.1872	217.1078	280.5131	334.2986	386.9797
21	10.0000	180.7733	232.5402	282.8423	343.8199	406.8986
22	10.5000	168.9519	215.8110	265.6914	303.2759	339.0953
23	11.0000	234.6301	288.6761	345.4504	400.8041	450.3278
24	11.5000	232.5068	276.4719	320.1711	348.0141	375.9461
25	12.0000	182.8282	232.6468	279.3274	330.2663	379.8887

Fuzzification of the Data

From an expert, we can obtain information about the **extreme value intervals** around $Y_d(t)$, let $[Y_d(t)_1, Y_d(t)_2]$, where $Y_d(t) \in [Y_d(t)_1, Y_d(t)_2] \subseteq [a(t), b(t)]$.



Fuzzy data "around $Y_d(t)$" for any time t.

Parameter Estimation with Fuzzy Data

As an initial illustration, the fuzzy data will be comparing by a simple population growth model, that is,

$$\begin{aligned} Y'(t) &= \lambda Y \\ Y(0) &= Y_0 \end{aligned}$$

; with the assumption that $\lambda \geq 0$.

Model Revision

- Y_0 is expressed as **fuzzy initial value**, that is $Y_0 = trimf\,(x, a_0, \bar{Y}_0, b_0)$
- $Y(t)$ is called by **fuzzy solution**, that is $Y(t) = trimf\,(x, a_1(t), \bar{Y}(t), b_1(t)), \forall$ t .
- Formed $\alpha - cut$ of both Y_0 and $Y(t)$, that are:

$[Y_0]^\alpha = \left[Y_{0\inf}^\alpha , Y_{0\sup}^\alpha \right]$ and $[Y(t)]^\alpha = \left[Y_{\inf}^\alpha (t), Y_{\sup}^\alpha (t) \right]$,

with $\alpha = mean\left\{ \operatorname{mod}\{\alpha_1(t)\}, \operatorname{mod}\{\alpha_2(t)\} \right\}, \forall t$.

The Set of *alpha-cut* Deterministic System

- Given the Equations (Seikkala 1987):

$$\begin{cases} \left(Y_{\inf}^{\alpha}\right)'(t) = \lambda Y_{\inf}^{\alpha}(t); \quad Y_{\inf}^{\alpha}(0) = Y_{0\inf}^{\alpha} \\ \left(Y_{\sup}^{\alpha}\right)'(t) = \lambda Y_{\sup}^{\alpha}(t); \quad Y_{\sup}^{\alpha}(0) = Y_{0\sup}^{\alpha} \end{cases} ;\alpha \in [0,1].$$

It is called by $\alpha - cut$ **deterministic systems**, and the solutions are:

$$\begin{cases} Y_{\inf}^{\alpha}(t) = Y_{0\inf}^{\alpha}\ e^{\lambda t} \\ Y_{\sup}^{\alpha}(t) = Y_{0\sup}^{\alpha}\ e^{\lambda t} \end{cases} ;\alpha \in [0,1], \lambda \geq 0$$

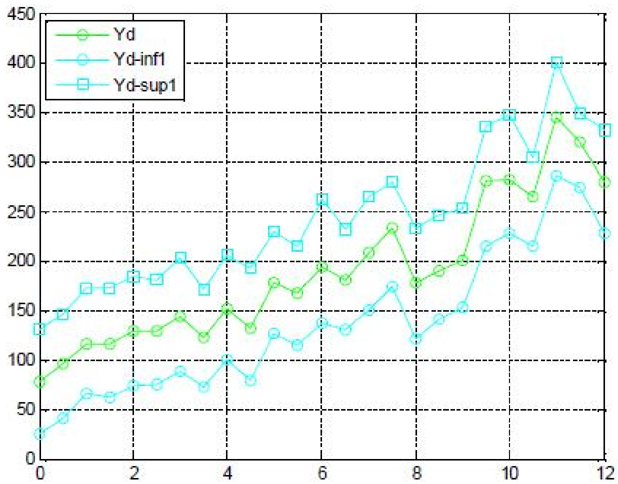
The solutions expresses an $\alpha - cut$ from fuzzy numbers $Y(t)$.

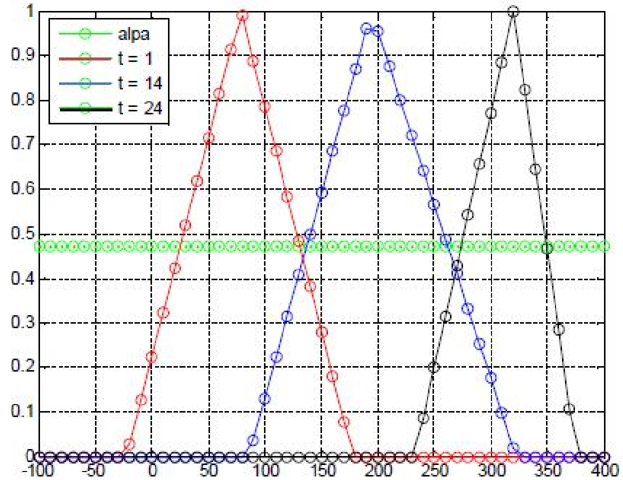
$$\alpha = mean\left\{\mathrm{mod}\{\alpha_1(t)\}, \mathrm{mod}\{\alpha_2(t)\}\right\}, \forall t, \qquad \Rightarrow \alpha = 0.4711$$

New significant values of the fuzzy data

The alpha value (0.4711) determines the new significant values of data fuzzy

t	$Yd\_inf1(t)$	$Yd(t)$	$Yd\_sup1(t)$
0	25.0767	78.9006	131.1647
0.5000	41.7802	96.7233	147.0172
1.0000	65.9176	116.7831	173.2555
1.5000	62.0153	116.1667	173.5343
2.0000	73.9055	129.7609	185.3273
2.5000	75.6713	130.0261	181.5455
3.0000	88.7877	144.1766	202.7409
3.5000	72.7415	123.6375	171.5296
4.0000	101.0983	152.4359	206.2530
4.5000	80.1625	132.2626	193.2284
5.0000	126.5831	177.7863	229.3043
5.5000	115.2140	167.9423	214.9955
6.0000	136.9230	194.1527	262.0993
6.5000	130.6254	181.2587	232.3358
7.0000	150.9872	207.9933	264.4981
7.5000	174.8661	234.0147	279.6612
8.0000	121.2261	178.4951	233.4697
8.5000	141.2559	190.2744	246.7245
9.0000	152.7000	200.8916	253.8172
9.5000	214.7581	280.5131	336.8225
10.0000	228.8588	282.8423	348.4547
10.5000	214.5266	265.6914	304.5142
11.0000	286.8384	345.4504	400.9192
11.5000	273.8061	320.1711	349.6701
12.0000	228.2897	279.3274	332.5135

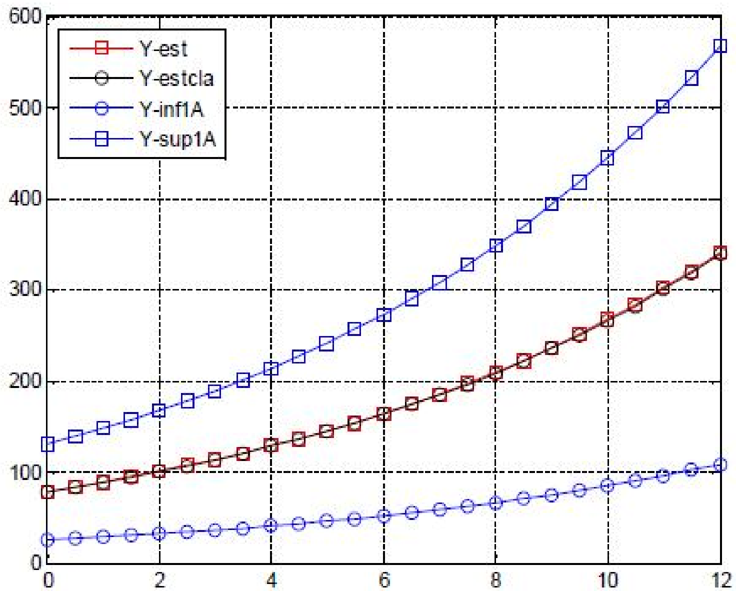


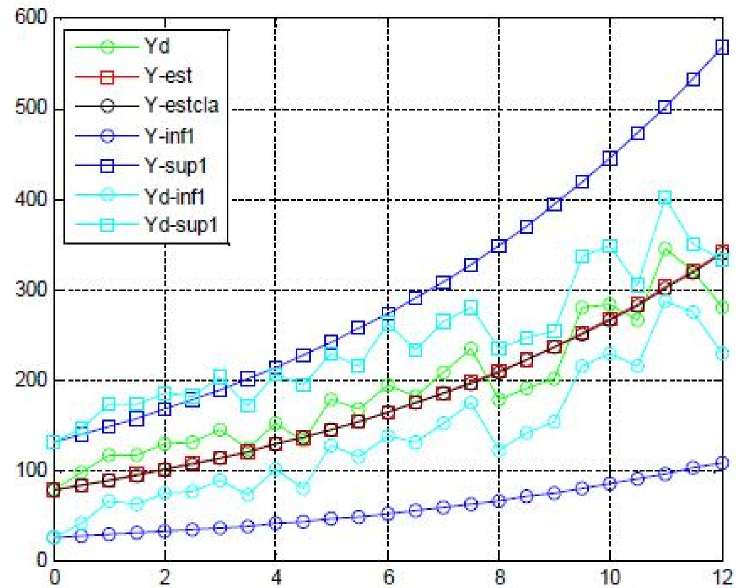


Parameter estimation is done by providing an initial estimate on the value of λ, ie $\lambda = 0.1$. Then obtained the value of the parameters

$$\lambda = 0.1220 \text{ and } \lambda_{cla} = 0.1214$$

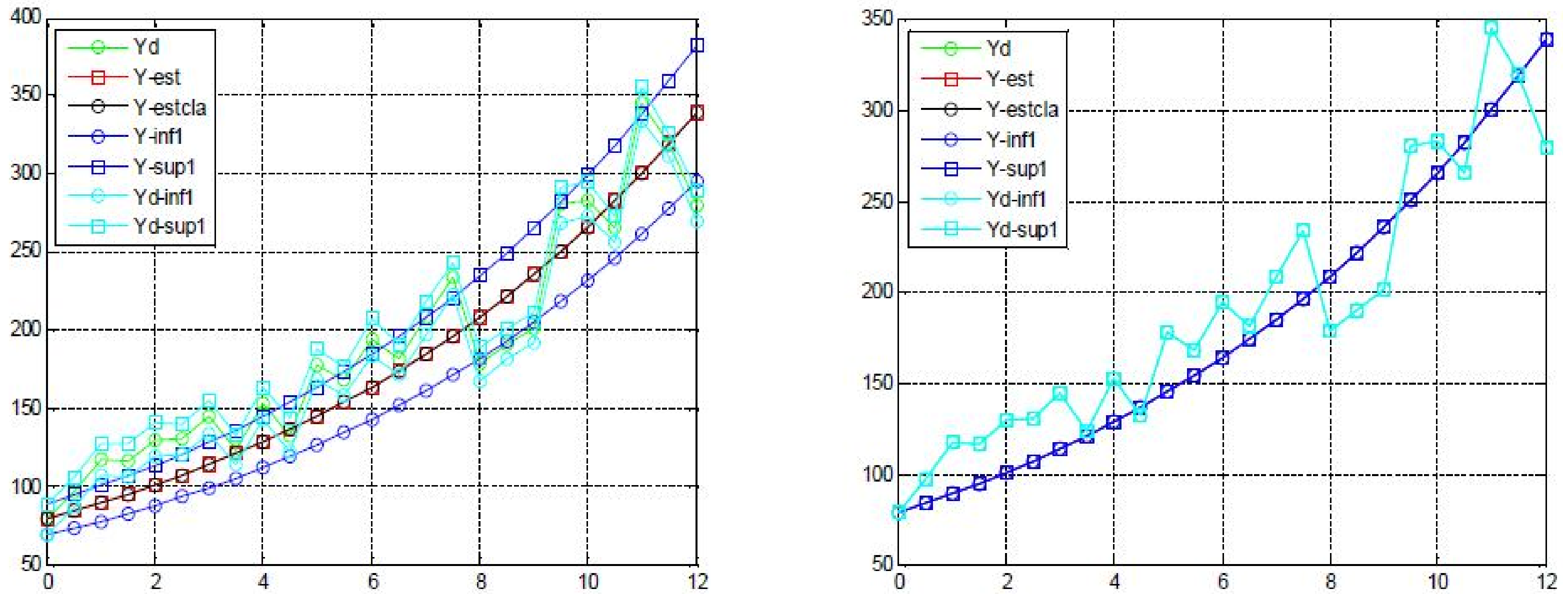
Using the initial values of *Yd*(0), *Yd*_inf1(0) and *Yd*_inf1(0), then obtained the following solutions:





Using the initial values of *Yd*(0), *Yd*_inf1(0) and *Yd*_inf1(0), by taking the values of α which is close to 1, i.e. 0.9 and 1, it is obtained, respectively:

$$\lambda = 0.1215 \text{ and } \lambda = \lambda_{cla} = 0.1214$$



- The value of alpha is inversely proportional to the uncertainty.

  If the alpha value is higher, then the uncertainty becomes lower, or, it can be said that the data more certain.

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